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Invited Comment

Quantum optics as a tool for photonic lattice design

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Abstract

We present the theoretical basis needed to work in the field of photonic lattices. We start by studying the field modes inside and outside a single waveguide. Then we use perturbation theory to deal with an array of coupled waveguides and construct a mode-coupling theory. Finally, we show how quantum optics models can be used as a toolbox to design photonic integrated circuits that behave as multiplexors, optical couplers, and optical oscillators.

Keywords: optical simulation, photonic lattices, quantum optics

1. Introduction

It was 1974 when Tsai and Thomas proposed the analogy between an optical waveguide coupler and the quantum-mechanical double well [1]. The ultimate goal for the simulation of physics was set up by Feynman in 1985—quantum simulators and universal quantum computers [2]—but classical optical analogs of quantum phenomena have also proved to be an impressive tool for carrying out studies with laboratory models that otherwise would belong only to the realm of theoretical physics [3]. In particular, arrays of more than two photonic waveguides have been a recursive example of classical simulators of quantum physics ever since the proposal of Christodoulides and Joseph to optically simulate nonlinear atomic chains [4].

The coupling of waveguides is an old problem in optical physics [5], and coupled mode theory is used to describe it [6–11]. The first versions of such photonic structures were oriented only toward their use in optical circuits [12], but modern fabrication techniques [13, 14] and a paradigm shift [4, 15] have cemented their use as classical simulators of quantum systems, which in turn is now aiding the design of integrated photonic devices [16].

In the following, we present a brief tutorial, including an extensive list of literature resources, that aims to introduce graduate students to the field of photonic lattices. The theoretical core is presented in the following two sections. First, the concept of normal modes of a waveguide is introduced by use of the Hertz potentials formalism. Then the basics of coupled mode theory are revisited. Later we discuss the experimental fabrication methods used to create these photonic structures, and finally, we explore how quantum optics can be used as a toolbox for the design of diverse photonic integrated circuits.

2. Modes in single waveguides

Waveguide structures support electromagnetic modes depending on their characteristics: geometry, dimensions, permittivity, permeability, etc. Let us solve the Maxwell equations for an infinitely large, isotropic, homogeneous dielectric medium that is invariant to translation over the z-axis in the absence of free charges. We will also suppose that the material that fills the waveguide is non-dissipative and non-magnetic, so the permittivity is real (and constant inside the waveguide, as we are also supposing that the medium is homogeneous) and the permeability is the one of vacuum μ_0. Usually we would write the differential sets for each component of the electromagnetic field [17], but here we want to bring forward an operator method based on Hertz potentials formalism [18] that may make things easier to understand. For axial symmetry, the electric field can be decomposed into transverse electric (TE) and magnetic (TM) components [19, 20]:

$$\vec{E} = \left\{ c_{TE}\left(\hat{e}_u \frac{1}{h_v}\frac{d}{dv} - \hat{e}_v \frac{1}{h_u}\frac{d}{du}\right) + c_{TM}\left[ik_z\left(\hat{e}_u\frac{1}{h_u}\frac{d}{du} + \hat{e}_v\frac{1}{h_v}\frac{d}{dv}\right) + \hat{e}_z k_\perp^2\right]\right\}\psi(\vec{r}), \quad (1)$$

where c_{TE} and c_{TM} are constants related to the transverse electric and transverse magnetic modes, respectively; the coordinate system (u, v, z) is given by the unit vectors $\hat{e}_j$ and the scaling factors by h_j; the wavevector $\vec{k}$ is given by its transverse, $k_\perp$, and axial, k_z, components such that $k^2 = k_\perp^2 + k_z^2$; and the Hertz potential answers to the scalar Helmholtz equation

$$\left(\nabla^2 + k^2\right)\psi(u, v, z) = 0. \quad (2)$$

The magnetic field can be calculated from Maxwell equations, their solution (1), and the constitutive equations $\vec{D} = \epsilon\vec{E}$ and $\vec{H} = \vec{B}/\mu$, with the permittivity and permeability of the material given by ϵ and μ, in that order.

Now there are four separable coordinate systems with axial symmetry, and each will have an associated scalar Hertz potential [21]; e.g., Cartesian coordinates will yield sinusoidal scalar potentials. If we assume a waveguide with polar symmetry in its transversal section, i.e., a circular cylinder with radius $\rho = a$ and infinite length, it will support modes related to the Hertz scalar potential

$$\psi_{in}(\vec{r}) = J_m(k_\perp\rho)e^{im\phi}e^{-ik_z z}e^{-i\omega t}, \quad (3)$$

where $J_m(x)$ are the Bessel functions of the first kind, with $m = 0, \pm 1, \pm 2, \ldots$. The transverse component of the wavevector is $k_\perp = \sqrt{\omega^2\mu\epsilon - k_z^2}$, with μ and ϵ the permeability and permittivity of the material, respectively. It is straightforward to show that the field modes defined by (1) and (3) form an orthogonal set that can be normalized if required. We cannot forget the modes supported by the vacuum outside the waveguide core, $\rho > a$, but first let us think about a given guided mode. For a guided mode, the power of the electric field propagates through the waveguide; in other words, $k_z \sim k = \omega\sqrt{\mu\epsilon}$. This means that if the velocity of light in the waveguide core, $\sqrt{\mu\epsilon}$, is slower than in the exterior, in this case $\sqrt{\mu_0\epsilon_0}$, then $k_z > \omega\sqrt{\mu\epsilon_0}$ and $k_\perp$ will be purely imaginary. Thus, we can write $\kappa_\perp = -ik_\perp$, and the scalar potential outside the waveguide is given by the expression

$$\psi_{out}(\vec{r}) = K_m(\kappa_\perp\rho)e^{im\phi}e^{-ik_z z}e^{-i\omega t}, \quad (4)$$

where we are using the modified Bessel functions of the second kind, $K_m(x)$, and we have to guarantee continuity of the entire electromagnetic field $\vec{E}_{in}(a) = \vec{E}_{out}(a)$ through an adequate choice of the constants c_{TE}^{in}, c_{TM}^{in}, c_{TE}^{out}, and c_{TM}^{out}; thus, we will have hybrid modes most of the time. The asymptotic behavior of the modified Bessel function, $K_m(x) \propto e^{-ix}$ for $x \gg 1$, leads to exponential decay of the scalar Hertz potential as $K_m(-ik_\perp\rho) \propto e^{-k_\perp\rho}$ for $\rho \gg a$. This implies that the field outside the waveguide core will exponentially decay with the distance from the boundary of the waveguide; i.e., it is an evanescent field. Thanks to this evanescent field, we expect that a second waveguide supporting an identical mode placed nearby may harvest the electrical field leaking from the first. In what follows, we will revisit coupled mode theory formalism to show this.

3. Mode coupling in arrays of waveguides

Let us consider an ideal array of waveguides without dissipation into the environment, and take one of them as the unperturbed waveguide and view the rest as perturbations [6–11]. The transversal forward propagating perturbed and unperturbed electric field modes at the nth waveguide are

$$\vec{E}_{n,\eta} = a_{n,\eta}(z)\vec{e}_n(x, y)e^{-i\omega t}, \quad \eta = p, u, \quad (5)$$

where the vector component $\vec{e}_n$ is one of the orthonormal transverse modes of the nth waveguide and the propagating field amplitude is such that for the unperturbed system $a_{n,u}(z) = e^{i\beta_n(z)}$ and for the perturbed system $a_{n,p}(z) = \mathcal{E}_n(z)$; we are going to use this notation because we will cycle through all the guides, treating one at a time as unperturbed. We can introduce the perturbation into the equations of Maxwell through the polarization field $\vec{P}$, such that $\vec{D} = \epsilon\vec{E} + \vec{P}$, so

$$\nabla \cdot \left(\epsilon_n\vec{E}_n + \vec{P}_n\right) = 0, \quad (6)$$

$$\nabla \cdot \vec{B}_n = 0, \quad (7)$$

$$\nabla \times \vec{E}_n = i\omega\mu_0\vec{H}_n, \quad (8)$$

$$\nabla \times \vec{H}_n = -i\omega\left(\epsilon_n\vec{E}_n + \vec{P}_n\right), \quad (9)$$

where the polarization field at the unperturbed waveguide is null, $\vec{P}_{n_0} = 0$. By use of all these definitions and some vector calculus identities, we can write Lorentz reciprocity between unperturbed and perturbed fields,

$$\int_S dS\,\nabla \cdot \left[\vec{E}_{n,p} \times \vec{H}_{n,u}^* + \vec{E}_{n,u}^* \times \vec{H}_{n,p}\right] = i\omega\int_S dS\,\vec{P}_n \cdot \vec{E}_{n,u}^*, \quad (10)$$

in the corresponding form,

$$\left(i\frac{d}{dz} + \beta(z)\right)\mathcal{E}_n(z)\int_S dS\left[\vec{e}_n \times \vec{h}_{n,m}^* + \vec{e}_n^* \times \vec{h}_{n,m}\right] \cdot \hat{e}_z = -\omega\int_S dS\vec{e}_{n_0,m}^* \cdot \vec{P}_n, \quad (11)$$

where S is an arbitrary surface [22–24]. The integral on the left-hand side in (11) can be identified with the power of the field modes after we revisit their orthonormality,

$$\int_S dS\left(\vec{e}_m \times \vec{h}_{\tilde{m}}^* + \vec{e}_{\tilde{m}}^* \times \vec{h}_m\right) = 2P_0\, \delta_{m,\tilde{m}}, \quad (12)$$

where the constant P_0 is the normal power of the field mode and the symbol $\delta_{a,b}$ stands for the delta of Kronecker. At this point, we need to make an assumption to simplify our work. We will assume that the differences between the waveguides are small; that is why we are treating them as perturbations. If the differences are small, the normal transversal modes in each waveguide are the same and we can write (11) in the form:

$$\left(i\frac{d}{dz} + \beta_n\right)a_n = -\frac{\omega}{4P_0}\int_S dS\vec{e}_n^* \cdot \vec{P}_n. \quad (13)$$

Now we need to determine what the right-hand side of the equation means. We can divide the contributions to the polarization field into those coming from the local and coupling perturbations,

$$\vec{P}_n = \vec{P}_{n,l} + \vec{P}_{n,c}, \quad (14)$$

with

$$\vec{P}_{n,l} = \Delta\epsilon_n \mathcal{E}(z)\vec{e}_n, \quad (15)$$

$$\vec{P}_{n,c} = \sum_{j=0}^{N-1} \Delta\epsilon_n a_j(z)\vec{e}_j, \quad (16)$$

where the functions $\Delta\epsilon_n$ are the deviations from the unperturbed permittivity ϵ. Note that we call $\vec{P}_{n,c}$ the contribution to the polarization field from the coupling due to it accounting for all the other waveguides in the system. Then we can define two contributions by substituting (14) into the left-hand side of (13):

$$\alpha_n = \frac{\omega}{2P_0}\int_S dS\Delta\epsilon_n \vec{e}_n^* \cdot \vec{e}_n, \quad (17)$$

$$c_{n,j} = \frac{\omega}{4P_0}\int_S dS\Delta\epsilon_n \vec{e}_n^* \cdot \vec{e}_j. \quad (18)$$

The first comes from the local polarization field and those terms with $j = n$ from the coupling polarization field. The other comes from the rest of the terms, $j \neq n$, of the coupling polarization field. Then we can describe the field dynamics in our array of coupled waveguides as the differential equation set,

$$\left(i\frac{d}{dz} + \beta_n + \alpha_n\right)\mathcal{E}_n + \sum_{j=0,j\neq n}^{N-1} c_{n,j}\mathcal{E}_j = 0. \quad (19)$$

If we are just interested in nearest neighbor coupling, which is when only the first neighbor waveguides are close enough to allow tunneling of light from/to one another, we can rewrite the differential set in the form [5]:

$$-i\frac{d}{dz}\mathcal{E}_0 = n_0(z)\mathcal{E}_0 + g_1(z)\mathcal{E}_1, \quad (20)$$

$$-i\frac{d}{dz}\mathcal{E}_1 = n_1(z)\mathcal{E}_1 + g_1(z)\mathcal{E}_0 + g_2(z)\mathcal{E}_2, \quad (21)$$

$$\vdots \quad (22)$$

$$-i\frac{d}{dz}\mathcal{E}_{N-2} = n_{N-2}(z)\mathcal{E}_{N-2} + g_{N-2}(z)\mathcal{E}_{N-3} + g_{N-1}(z)\mathcal{E}_{N-1}, \quad (23)$$

$$-i\frac{d}{dz}\mathcal{E}_{N-1} = n_{N-1}(z)\mathcal{E}_{N-1} + g_{N-1}(z)\mathcal{E}_{N-2}. \quad (24)$$

We will refer to the parameter $n_j(z)$ as the effective refractive index due to its relation to the permeability and its perturbations in the jth waveguide and to the parameter $g_j(z)$ as first neighbor couplings. If we need to deal with kth neighbor couplings, we can do it starting from (19).

4. Experimental fabrication

The first array of optical waveguides reported in the literature was fabricated by proton implantation in GaAs [12, 25]. Later the development of high-energy femtosecond pulse lasers prompted the study of photochemical reactions in different glasses for their use in integrated optical circuits. In short, when a high-intensity ultra-short laser pulse is focused inside a material, absorption occurs and a microplasma is formed due to optical breakdown which induces permanent changes in the refractive index of the damaged area [13]. These changes are highly reproducible, and by moving the sample or the focused laser, waveguides can be written in bulk materials, e.g., fused and doped silica [13, 14, 26–28]. Typically, a high-energy pulsed Ti:Sapphire laser is used to process the glass material, e.g., 800 nm with power in the hundreds of mW for fused silica. The laser light is focused through an optical system that scans and damages the material at a given rate and can create low-loss three-dimensional photonic structures. This process has also been shown to reduce the nonlinear refractive index of written waveguides [4, 29, 30]. It is also possible to create waveguide arrays susceptible to real-time modifications by use of thermo-optic materials [31].

5. Quantum optics as a design toolbox

Notice that if we complex conjugate (24), it can be written in the short form

$$\frac{d}{dz}\mathbf{E}^* = -i\mathbf{H}\mathbf{E}^*, \quad (25)$$

where the asterisk stands for complex conjugation and $\mathbf{H}$ is a real symmetric tridiagonal matrix. Thus, it has the form of a Schrödinger equation where the conjugated field amplitudes play the role of the wavefunction coefficients and the propagation variable that of time. This allows us to use all knowledge of quantum optics as a toolbox for photonic lattice design because, as Feynman stated in his lectures, ‘the same equations have the same solutions’.

Note that in the case of a matrix $\mathbf{H}$ that does not depend on the propagation distance z, the solution to this vector differential equation is

$$\mathbf{E}^*(z) = e^{-i\mathbf{H}z}\mathbf{E}^*(0), \tag{26}$$

where the vector $\mathbf{E}(0)$ stores the initial field amplitudes impinging on the array of coupled waveguides. When the matrix is z-dependent, $\mathbf{H}(z)$, numerical analysis techniques like Runge–Kutta methods can be employed to solve the differential set. Let us consider a few examples in the following. Note that detecting the intensity at each of the waveguides is the simplest experimental measure, and one must look for the variables that are related to the intensities because measuring relative phases is typically cumbersome.

5.1. Nonclassical states of light for random walks (loaded multiplexors)

Imagine the simplest quantum walk, where at each step you have a fifty-fifty chance of walking left or right and the probability of finding the walker here or there is given by quantum interference of the paths. That is equivalent to an electromagnetic field propagating through a one-dimensional array of identical waveguides homogeneously coupled by nearest neighbor interactions [5, 12, 76, 77]. The variation of the coupling parameters directly affects the left/right walking probability, as also do varying refractive indices. We can use our knowledge of nonclassical states of light to generate specific walks. For example, a well-known nonclassical state of light is displaced number states [32], which we can write as

$$|m, igz\rangle = e^{igz\left(\hat{a}^\dagger + \hat{a}\right)}|m\rangle, \tag{27}$$

with the imaginary coherent parameter igz; the operator $\hat{a}$ ($\hat{a}^\dagger$) annihilates (creates) photons for a mode of the quantum field represented in the number basis, $|m\rangle$. Then, we can note that (32) solves the differential equation,

$$\frac{d\mathbf{E}(z)}{dz} = ig\left(\hat{a}^\dagger + \hat{a}\right)\mathbf{E}(z). \tag{28}$$

In matrix form, the annihilation and creation operators have elements $a_{m,n} = \sqrt{m}\,\delta_{m+1,m}$ and $a^\dagger_{m,n} = \sqrt{m}\,\delta_{m,m+1}$ with $m = 0, 1, \ldots$, such that we can write the preceding differential equation as the photonic lattice composed of identical waveguides:

$$-\imath\frac{d\mathcal{E}_0(z)}{dz} = g\mathcal{E}_1(z), \tag{29}$$

$$-\imath\frac{d\mathcal{E}_j(z)}{dz} = g\left[\sqrt{j}\,\mathcal{E}_{j+1}(z) + \sqrt{j-1}\,\mathcal{E}_{j-1}(z)\right], \tag{30}$$

$$-\imath\frac{d\mathcal{E}_{N-1}(z)}{dz} = g\sqrt{N-2}\,\mathcal{E}_{N-2}(z). \tag{31}$$

Such a photonic lattice has been called a Glauber–Fock photonic lattice in the literature [33–38]. If we follow this scheme, we can also design photonic integrated circuits that provide us with intensity distributions related to squeezed states:

$$|m, igz\rangle = e^{igz\left(\hat{a}^{\dagger 2} + \hat{a}^2\right)}|m\rangle. \tag{32}$$

Squeezed states solve the following differential equation set:

$$\frac{d\mathbf{E}(z)}{dz} = i\left(\hat{a}^{\dagger 2} + \hat{a}^2\right)\mathbf{E}(z), \tag{33}$$

and this can be written as two uncoupled photonic lattices [39]. Of course, this approach can be extended to j uncoupled photonic lattices for couplings of the form $\left(\hat{a}^{\dagger j} + \hat{a}^j\right)$. Recently it has been shown that the creation of other classical analogs of nonclassical states, such as W-states [40, 41] and squeezed states [42], is possible in arrays of one- and two-dimensional waveguides [79]. Furthermore, by using intensity correlation schemes like the Hanbury Brown–Twiss correlations, it is possible to study the path entanglement created by propagation of photonic lattices [43, 44]. There will come a time when multiple single photons propagating through optical waveguide arrays will be used to generate proper quantum random walks with high-order correlation functions[78, 80].

5.2. Three-level atoms driven by classical fields for optical couplers

The dynamics of the population amplitudes of a class of three-level atom driven by two classical fields, where the refractive index is equivalent to the role of the self-energies of the atomic states and the couplings are the analog of time-dependent classical driving fields coupling the atomic states, is equivalent to the differential set for three non-identical waveguides coupled between them,

$$\frac{d\mathbf{E}(z)}{dz} = i\begin{pmatrix} n_1(z) & \alpha(z) & 0 \\ \alpha(z) & n_2(z) & \beta(z) \\ 0 & \beta(z) & n_3(z) \end{pmatrix}\mathbf{E}(z). \tag{34}$$

This allows us to use all the machinery developed in quantum optics to construct propagators based on the Lie group generators of $su(3)$ [45]. At this point, we can use quantum optical phenomena for photonic circuitry purposes, e.g., one-directional adiabatic three-waveguide couplers [46–54] that can be considered the optical analog of stimulated Raman adiabatic passage (STIRAP) [55]. As an example, we can set $n_j = 0$, $\alpha(z) = ce^{-(z-\zeta)^2/\zeta^2}$, and $\beta(z) = ce^{-z^2/\zeta^2}$ where $c, \zeta \in \mathbb{R}$ [56] and obtain perfect transfer from the first to the last waveguide. We can also use the parameter set $\alpha(z) = \beta(z) = c$, $\gamma(z) = -m\cos(z)/2$, and $\delta(z) = \sqrt{3}\gamma(z)$ with $c, m \in \mathbb{R}$ related to the so-called atomic population trapping [57] whenever the value of the constant m produces a zero in the zeroth-order Bessel function, $J_0(m) = 0$. Thus, we can use this to produce coherent oscillations of the field amplitudes through the photonic device that deliver intensity trapping at certain intervals for a finite propagation distance [58].

5.3. A two-level atom coupled to a quantum field for optical oscillators

The Jaynes–Cummings (JC) model is a work horse in quantum optics. The inclusion of so-called counter-rotating terms, arbitrary couplings, and nonlinear processes in the basic JC model may not be feasible in quantum optics experiments,

$$\hat{H} = h(\hat{n}) + \frac{\omega_0}{2}\hat{\sigma}_z + g_-\left(\hat{a}\frac{f(\hat{n})}{\sqrt{\hat{n}}}\hat{\sigma}_+ + \frac{f(\hat{n})}{\sqrt{\hat{n}}}\hat{a}^\dagger\hat{\sigma}_-\right) + g_+\left(\hat{a}\frac{f(\hat{n})}{\sqrt{\hat{n}}}\hat{\sigma}_- + \frac{f(\hat{n})}{\sqrt{\hat{n}}}\hat{a}^\dagger\hat{\sigma}_+\right), \quad (35)$$

where the real functions $f(\hat{n})$, $g(\hat{n})$ and $h(\hat{n})$ can be nonlinear functions of the number operator, $\hat{n} = \hat{a}^\dagger\hat{a}$, and the parameters ω_0 and $g_\pm$ are the qubit frequency, rotating couplings, and counter-rotating couplings, in that order. This model describes a plethora of qubit–field interactions [59–63] and can be easily set in the short form (25) by using a parity decomposition, $|\psi_\pm\rangle = \sum_j \mathcal{E}_j^{\pm}(z)|\pm, j\rangle$, with $|+(-), j\rangle = \hat{\sigma}_x^j\,|g(e), j\rangle$, plus the matrix representation of Pauli matrices and bosonic operators used before [59],

$$i\frac{d}{dz}\mathcal{E}_0^{(\pm)} = d^{(\pm)}(0)\mathcal{E}_0^{(\pm)} + g_\pm f(1)\mathcal{E}_1^{(\pm)}, \quad (36)$$

$$i\frac{d}{dz}\mathcal{E}_{2k+1}^{(\pm)} = d^{(\pm)}(2k+1)\mathcal{E}_{2k+1}^{(\pm)} + g_\pm f(2k+1)\mathcal{E}_{2k}^{(\pm)} + g_\mp f(2k+2)\mathcal{E}_{2k+2}^{(\pm)}, \quad k \geqslant 0, \quad (37)$$

$$i\frac{d}{dz}\mathcal{E}_{2k}^{(\pm)} = d^{(\pm)}(2k)\mathcal{E}_{2k}^{(\pm)} + g_\mp f(2k)\mathcal{E}_{2k-1}^{(\pm)} + g_\pm f(2k+1)\mathcal{E}_{2k+1}^{(\pm)}, \quad k \geqslant 1, \quad (38)$$

with $d^{(\pm)}(j) = h(j) \mp (-1)^j\frac{\omega_0}{2}$. Thus, we end up with two one-dimensional arrays of coupled waveguides, each describing a parity subspace of the qubit–field system. Just to give an example of an application, these photonic lattices can be used to generate complicated optical oscillators [59–63].

Furthermore, the Majorana equation [64, 65], $\imath\hbar\gamma^\mu\partial_\mu\psi = m\psi_c$, in (1+1) dimensions, can be reduced to two Dirac equations for Majorana fermions with opposite mass signs [66],

$$\imath\hbar\partial_t\phi_\pm = \left(c\hat{p}_x\hat{\sigma}_x \pm mc^2\hat{\sigma}_z\right)\phi_\pm, \quad (39)$$

for the Majorana fermions $\phi_\pm \in \mathbb{C}^2$, i.e., $\phi_\pm = \hat{\sigma}_z\hat{\sigma}_x\phi_\pm^*$. Here we can borrow the definition of the linear momentum operator in terms of the creation/annihilation operators, $\hat{p}_x = (\hat{a}^\dagger + \hat{a})/\sqrt{2}$, and realize that it is identical to (35) by setting $\hbar = 1$, $h(\hat{n}) = 0$, $\omega_0 = \pm 2mc^2$, $f(\hat{n}) = \sqrt{n}$, and $g_+ = g_- = c/\sqrt{2}$ [67]. Thus, we can use our knowledge of Majorana physics to design optical analogs of relativistic oscillators and propagators. The Dirac equation has already been studied in relation to binary lattices [68–71]. We can use Zitterbewegung to generate an optical lattice where the barycenter of the field intensity oscillates as it propagates [72] or use Klein tunneling to generate controlled reflection of light at the interface between two photonic lattices [73–75].

6. Conclusions

Quantum optics, or quantum mechanics in general, may help us design optical lattices that fulfill diverse roles, e.g., waveguide couplers [46–55, 58], multiplexors [40, 41], light rectifiers [112], and on-demand sources of tailored guided modes [113]. Some other known optical lattices, related to quantum mechanical phenomena, involve diffraction such as multiband diffraction and refraction [114], generation of surface waves [115–118] and light bullets in nonlinear periodically curved waveguides [119], the acceleration of Wannier–Stark states in uniform optical lattices [120], and generation of two-dimensional Airy-like [121] and other structures [122, 123], with the goal of controlling light propagation [124–126]. Some structures can provide self-focusing [4], others perfect imaging [127], and others transport control through the phase of the impinging fields [128]. It also is possible to use supersymmetric quantum mechanics to design isospectral one-dimensional crystals [129–131] or produce resonant propagation via local $\mathcal{PT}$ invariance [132]. Two-dimensional structures can simulate effective electromagnetic fields for bosons [133, 134]. The use of waveguide arrays combining gain and loss allows us to classically explore the Hubbard model [135], highly correlated states [136], and non-Hermitian $\mathcal{PT}$-symmetric systems [137–143]. If we include our knowledge of electron propagation in structures, we can realize Bloch oscillations [81–83], Bloch–Zenner oscillations [84–86, 86–89], coherent tunneling [1, 90], and the Zeno effect [54, 91–93], as well as dynamic [102, 106, 107, 105] and Anderson localization [54, 94–101, 101–104], to mention a few. Results from graphene physics can be used to design honeycomb lattices with helical modulation that deliver dynamics equivalent to band collapse [108], and topological insulators [109–111].

A number of factors arise with respect to the challenge that is the control of transport of light in optical lattices. Of course, there surely exists a trove of quantum systems that have not been explored yet, as any given quantum system that can be written in terms of a real symmetric matrix is feasible for classical simulation with nearest neighbor arrays. Actually, more complex systems with gain and loss [132] or 2D structures [133, 134] allow us to increase the range of quantum models feasible for simulation. Plus, the experimental simulation of systems that analytically break, e.g., those with SU(1,1) symmetry, should be really interesting for studying the transition from unitarian to non-unitarian dynamics in theoretical physics [59]. At the moment, we try to focus on real-world applications by using these classical simulations as building blocks for the design of photonic integrated circuits. We are also interested in the propagation of photons through optical lattices for quantum communications and processing of information, as photonic structures integrated with single-photon sources will be a common occurrence in the near future [144–146].

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