Optics Communications 356 (2015) 170–174

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optimal focusing of a beam in a ring vortex

Victor Arrizón*, Ulises Ruiz, Dilia Aguirre-Olivas, Gabriel Mellado-Villaseñor

Instituto Nacional de Astrofísica, Óptica y Electrónica, Puebla 72000, México 20036 Mexico

ARTICLE INFO

Article history:
Received 7 May 2015
Received in revised form
15 July 2015
Accepted 17 July 2015
Available online 3 August 2015

Keywords:
Optical vortices
Computer holography
Diffractive optics

ABSTRACT

Conventional light focusing, i.e. concentration of an extended optical field within a small area around a point, is a frequently used process in Optics. An important extension to conventional focusing is the generation of the annular focal field of an optical beam. We discuss a simple optical setup that achieves this kind of focusing employing a phase plate as unique optical component. It is assumed that the annular focal field is modulated by an azimuthal phase of integer order q that converts the field in a ring vortex. We first establish the class of beams that being transmitted through the phase plate can be focused into a ring vortex. Then, for each beam in this class we determine the plate transmittance that generates the vortex with the maximum possible intensity, which is referred to as optimal ring vortex.

1. Introduction

Light focusing, one of the processes more employed in optics, is usually realized by a lens. An infinitely small focal point cannot be achieved in free space beam propagation. Even with large aperture lenses, and infinitely extended beams, the minimum focal field size is in the order of half the wavelength [1]. Here we discuss focusing of monochromatic light in an annular focal field, assuming that it is modulated by an azimuthal phase of arbitrary integer order q. The inclusion of the topological charge transforms the focal field in a ring vortex (RV). This type of structured field can be useful in several applications, e.g. optical trapping with orbital angular momentum transference [2–4], lithography [5,6], high-resolution fluorescence microscopy [7], quantum entanglement [8–10], and vortex coronography [11,12].

As occurs in conventional focusing, the generation of an infinitely narrow RV [13–16] is impossible. Therefore, it is important to establish the optimal approximation to this field that can be physically implemented. We consider that the optimal RV generated by a given optical beam is the one with the maximum possible intensity. The maximum intensity in RVs implies other attributes, as narrow transverse section and high intensity gradient that may offer advantages in different applications of such fields.

The generation of an optimal RV at the Fourier domain of a phase diffractive element, which is illuminated by a Gaussian beam (GB), has been recently reported [17]. In the present communication we discuss the simplest method for annular focusing, with arbitrary integer order topological charge, of an input beam. This method employs a phase plate as unique optical component, which modulates the complex amplitude of the beam. The RV is obtained, by free propagation of the modulated beam, at a specific distance from the plate. In Section 2, as first step, we establish the class of beams that can generate a RV using this simple method. Then, we determine the phase plate transmittance required to achieve the optimal annular focusing of the beams in this class. In Section 3 we illustrate the features of optimal RVs, employing both numerical simulations and experiments. In Section 4, we present final remarks and conclusions.

2. Theory

To discuss annular focusing of a beam we refer to the setup depicted in Fig. 1. In this setup, the input beam (B) is passed through a phase plate (PP), and the RV is generated, by free propagation of the transmitted beam, at a distance z from the plate.

For our analysis, the optical fields are expressed in polar coordinates (ξ,ϕ) at the plate plane and (r,θ) at the focal field plane. The complex amplitude of a generic RV, with integer topological charge q, is

$$h(r,\theta) = F(r)\exp(iq\theta), \tag{1}$$

whose radial factor $F(r)$ is specified below. Considering that the RV, with the separable form in Eq. (1), is obtained by free propagation of the field transmitted by the plate, it is easy to prove that this field must have the separable form

$$f(\xi,\phi) = a(\xi)\exp[i\beta(\xi)]\exp(iq\phi), \tag{2}$$

where the amplitude $a(\xi)$ is a non-negative function and $\beta(\xi)$ is a radial phase function to be determined. This result is obtained

* Corresponding author.
E-mail address: arrizon@inaoep.mx (V. Arrizón).

http://dx.doi.org/10.1016/j.optcom.2015.07.049
0030-4018/

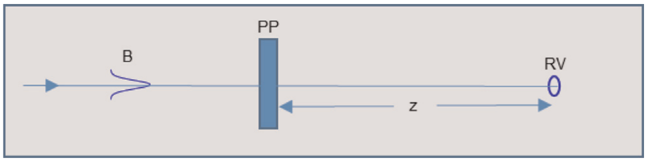

Fig. 1. Simple setup to generate the annular focusing of a beam.

using either the exact or the paraxial scalar field propagation.

Now we assume that the complex amplitude of the beam that illuminates the phase plate is $g(\xi,\phi)=|g(\xi,\phi)|\exp[i\alpha(\xi,\phi)]$, with amplitude $|g(\xi,\phi)|$ and phase $\alpha(\xi,\phi)$. Thus, denoting the phase plate transmittance as $p(\xi,\phi)$, we establish the identity $f(\xi,\phi)=g(\xi,\phi)\, p(\xi,\phi)$. Expressing this relation considering Eq. (2) and the previous formula for $g(\xi,\phi)$ it is easy to show that the complex amplitude of the required input beam is

$$g(\xi,\phi) = a(\xi)\exp[i\alpha(\xi,\phi)], \tag{3}$$

and the transmittance of the phase plate is given by

$$p(\xi,\phi) = \exp[i\beta(\xi)]\exp[-i\alpha(\xi,\phi)]\exp(iq\phi). \tag{4}$$

According to Eq. (3), the family of beams that can be transformed, using the setup of Fig. 1, into the ring vortex with the separable form of Eq. (1), must have an amplitude dependent only in the radial coordinate ξ. However, the phase $\alpha(\xi,\phi)$ in such beams can be an arbitrary function.

The unknown phase $\beta(\xi)$ in Eq. (4) is next specified in order to give desired attributes to the radial factor, $F(r)$, of the RV field. Performing the Fresnel propagation of the field $f(\xi,\phi)$ [Eq. (2)] to a distance z one obtains the field $h(r,\theta)$ [Eq. (1)], whose radial factor is (omitting constants)

$$F(r) = \exp\left(i\frac{kr^2}{2z}\right)\int_0^\infty \xi a(\xi)\exp\left[i\left(\beta(\xi)+\frac{k\xi^2}{2z}\right)\right]J_q\left(2\pi\frac{r}{\lambda z}\xi\right)d\xi, \tag{5}$$

where $k=2\pi/\lambda$ is the wave number and J_q denotes the q-*th* order Bessel function of the first kind. The integral in Eq. (5) corresponds to the q-th order Hankel transform of the radial function $a(\xi)\exp\{i[\beta(\xi)+k\xi^2/2z]\}$.

Now, let us assume that we desire a RV with radius r_0. We determine the radial phase $\beta(\xi)$ for which this focal field is optimum. From Eq. (5) we can establish the RV intensity at $r=r_0$ as

$$|F(r_0)|^2 = \left|\int_0^\infty f_{pos}(\xi)\exp\left[i\left(\beta(\xi)+\frac{k\xi^2}{2z}\right)\right]\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}d\xi\right|^2, \tag{6}$$

where $f_{pos}(\xi)=\xi\, a(\xi)|J_q(2\pi r_0\xi/\lambda z)|$ is a non-negative real function, and ‘sgn$\{x\}$’ is a binary function, equal to $+1$ for $x\geq 0$, and -1 otherwise. Since the integrand in Eq. (6) is formed by the non-negative function $f_{pos}(\xi)$ multiplied by phase factors, we can obtain the relation [18]

$$|F(r_0)|^2 \leq \left(\int_0^\infty f_{pos}(\xi)d\xi\right)^2, \tag{7}$$

where the squared integral represents the upper bound value for $|F(r_0)|^2$. It is straightforward to establish from Eq. (6) that the intensity $|F(r_0)|^2$ achieves the upper bound value if

$$\exp[i\beta(\xi)] = \exp\left(-i\frac{k\xi^2}{2z}\right)\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}. \tag{8}$$

Considering this result in Eq. (4) one obtains the plate phase modulation that generates the optimal RV of radius r_0, which is given by

$$p(\xi,\phi) = \exp\left(-i\frac{k\xi^2}{2z}\right)\exp[-i\alpha(\xi,\phi)]\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}\exp(iq\phi). \tag{9}$$

The phase plate with the transmittance in Eq. (9), illuminated by the input beam $g(\xi,\phi)$ [Eq. (3)], transmits the field $f(\xi,\phi)=a(\xi)\exp(-ik\xi^2/2z)\, \mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}\exp(iq\phi)$. Because of the quadratic phase factor in $f(\xi,\phi)$, the complex amplitude of the RV, at the distance z from the plate, is equivalent to the Fourier transform of the function $a(\xi)\, \mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}\exp(iq\phi)$.

An important input field that belongs to the class defined in Eq. (3) is the GB, whose complex amplitude can be expressed, omitting factors that are independent of ξ, as

$$g(\xi,\phi) = \exp(-\xi^2/w^2)\exp(ik\xi^2/2R), \tag{10}$$

where w is the beam radius and R is the curvature radius of the quadratic phase, at the plate plane. In order to apply the general results to the case of the input GB, it is required to replace the amplitude and the phase in Eq. (3) by $\exp(-\xi^2/w^2)$, and $k\xi^2/2R$, respectively. Thus, the plate transmittance that transforms the input GB in an optimal RV, with topological charge q, is

$$p(\xi,\phi) = \exp\left\{-i\frac{k}{2}\left(\frac{1}{R}+\frac{1}{z}\right)\xi^2\right\}\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}\exp(iq\phi). \tag{11}$$

Note that the quadratic phase factor in Eq. (11), corresponds to the transmittance of a conventional lens, which generates the Fourier transform of the other two factors. On the other hand, the annular form of the focal field, with maximum peak intensity, is allowed by the radial binary phase modulation $\mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}$. This last factor, together with the azimuthal phase, in both Eq. (9) and Eq. (11), correspond to the phase modulation of the q-th order Bessel beam of radial spatial frequency $\rho_0=r_0/\lambda z$. Two illustrative examples of the phase modulation in Eq. (11), with topological charges $q=0$ and $q=1$, respectively, are depicted in Fig. 2.

Our discussion and results are connected with previous works dealing with the so called perfect vortex [13–16], which is an infinitely narrow RV with arbitrary integer topological charge. It is clear that this ideal field cannot be generated in practice. However, the optimal physically realizable approximation to such ideal RV, employing the optical setup in Fig. 1, is generated by the phase plate whose transmittance is given by Eq. (9), for a generic beam with the complex amplitude specified in Eq. (3), or by Eq. (11), for an input GB. Such phase transmittances can be, in principle, fabricated by lithography on a glass substrate. An attractive option, discussed below, is the use of a phase liquid-crystal (LC) spatial light modulator (SLM).

3. Computational and experimental assessment of optimal RV generators

Next we develop numerical simulations to evaluate optimal

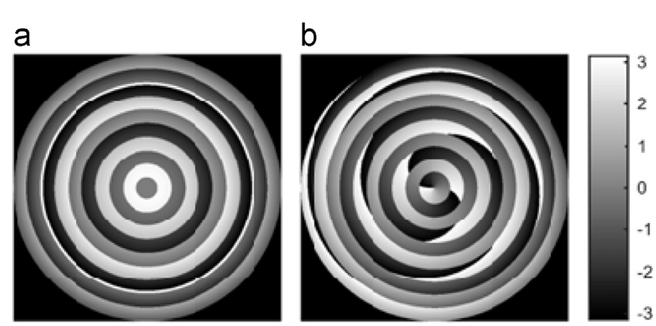

Fig. 2. Central sections in phase modulations of phase plates that generate optimal RVs of topological charges (a) $q=0$, and (b) $q=1$, employing an input Gaussian beam.

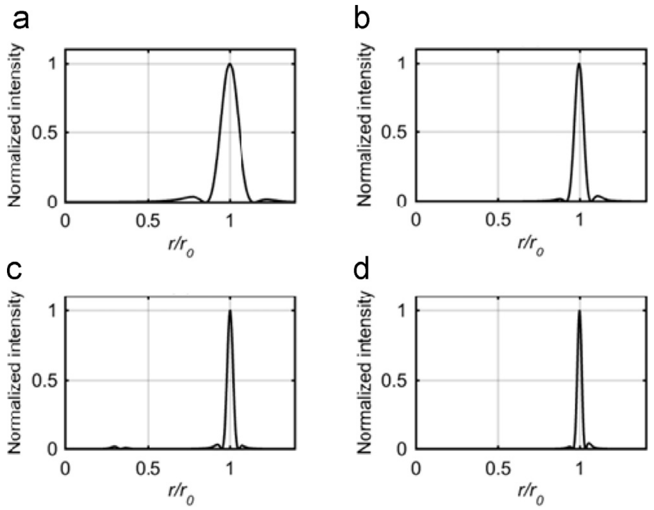

Fig. 3. Transverse intensity profiles of optimal RVs of topological charge $q=0$, generated with input Gaussian beams of width $w=Q\lambda z/r_0$, with parameter Q equal to (a) 2.5, (b) 5, (c) 7.5, and (d) 10.

RVs generated by an input GB. We assume that the GB radius w is related with the RV radius r_0, by the relation $w=Q\lambda z/r_0$, for different values of Q. In addition, for simplicity we assume an infinite curvature radius R in the quadratic phase of the GB. The normalized transverse intensity profiles of the generated RVs, for $Q=2.5$, 5, 7.5, and 10 and the topological charge $q=0$, are depicted in Fig. 3. We note that the width of the RV bright section, relative to the radius r_0, varies inversely with the value of the parameter Q. This result is understood noting that the radius w of the input GB is proportional to Q and considering that the transverse section of the RV is related to the Fourier transform of the GB profile.

The RV intensity profiles in Fig. 3 were computed for $q=0$. However, by means of additional simulations we obtain that the peak intensities and widths of optimal RVs (generated by GBs) show very low variation when Q is fixed and the topological charge q takes different values. We computed the peak intensity and the full width at half of the maximum intensity (FWHM) in RVs obtained for different values of Q and q. In Fig. 4 we display the peak intensities (a, b) and FWHM's (c, d) for $Q=5$ (a, c) and $Q=10$ (b, d). The peak intensities for different q's and a fixed Q are normalized respect to the peak intensity for $q=0$, and the FWHM's are normalized with respect to r_0.

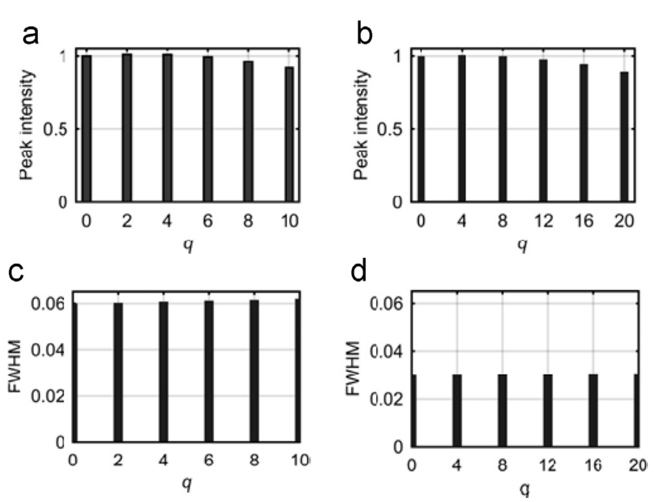

Fig. 4. Normalized peak intensities (top) and FWHMs (bottom) of optimal RVs for several topological charges q, generated by a GB with parameter Q equal to 5 (a,c) and 10 (b,d).

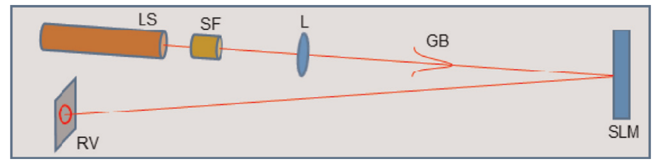

Fig. 5. Optical setup to generate the optimal annular focusing of a GB using a phase SLM.

In general we have found that for a fixed Q, the peak intensities (and corresponding FWHM's) of optimal RVs, present a relatively low variance for topological charges q in the range [0, 2Q]. The q-values for each plot in Fig. 4 correspond to this range.

For the generation of an optimal RV, with the described approach, we employ the experimental setup depicted in Fig. 5. In this setup, the light beam from a He–Ne laser source (LS) is cleaned and expanded by a spatial filter (SF), and collimated by a lens (L). It generates a GB that illuminates a reflecting phase SLM (Model PLUTO, Holoeye Photonics LG). In the SLM we implemented the phase for optimal annular focusing of the GB [Eq. (11)] with an additional linear phase modulation, whose purpose is to separate the focal field from the un-modulated fraction of the light reflected by the SLM. The intensity of RVs is recorded by a CCD camera.

In the experiment we employed a GB, obtained from a He–Ne laser ($\lambda=632.8$ nm). The measured beam parameters at the SLM plane were $w\cong 992$ µm and $R\cong 8.8$ m. For these parameters, we designed and tested phase plates, with the transmittance given in Eq. (11), considering the focal distance $z=50$ cm, and two different values of the RV radius r_0, which are computed from the relation $r_0=Q\lambda z/w$, for Q=2.5 and $Q=5$. The values for r_0 are 797 µm (for $Q=2.5$) and 1594 µm (for $Q=5$). The linear phase modulation introduced in the SLM transmittance is given by $\exp[i2\pi u_0(x+y)]$, with spatial frequency u_0 equal to 1/6 of the SLM bandwidth (the inverse of the SLM pixel resolution). In Fig. 6 we display a typical image of a generated RV, together with the 0-th order light spot. This spot basically results from the unmodulated reflection of the Gaussian beam that illuminates the SLM. In Fig. 7(a, b) we show the intensities of the RVs recorded by the CCD that are obtained for the case $q=0$. In Fig. 8 we display the transverse intensity profiles of the generated RVs vs the normalized radial coordinate r/r_0. The experimental ring intensity images were registered using the full CCD dynamic range (with 256 Gy levels). Thus, the background noise, which presents gray levels from 15 to 20, is not easily noted at Fig. 7. On the other hand, in the transverse profiles of Fig. 8, we have subtracted the minimum background intensity level, which corresponds to the gray level 15. The results obtained when we change the topological charge q (in the range [0, 2Q]) are quite similar to the ones for $q=0$. The experimental RV transverse profiles in Fig. 8 are highly coincident with the RV profiles in Fig. 3 (a, b), which were numerically computed for the parameters $Q=2.5$ and $Q=5$. In order to obtain high symmetry RVs it was

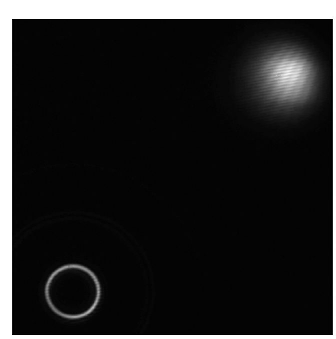

Fig. 6. Image of a typical generated RV, together with the 0-th order light spot.

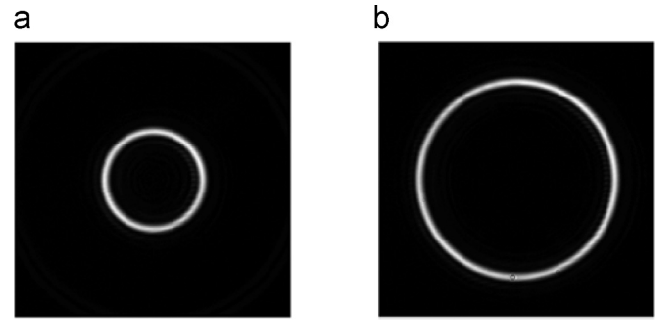

Fig. 7. Experimentally recorded intensities of optimal RVs with topological charge $q=0$ and parameter Q equal to (a) 2.5 and (b) 5.

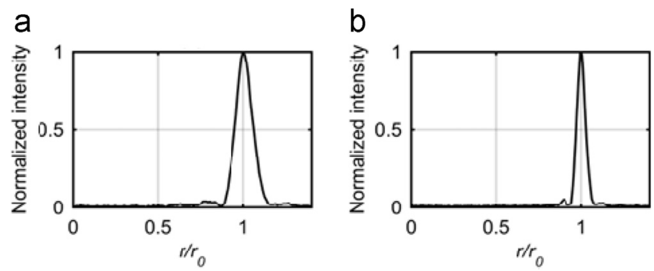

Fig. 8. Transverse intensity profiles of the experimentally generated RVs depicted in Fig. 6.

required the precise centering of this beam in the phase pattern displayed at the SLM. We achieve it using a conventional micrometric positioning system to control the transverse position of the SLM.

4. Final remarks and conclusions

The presence of the binary phase modulation $sgn\{J_q(2\pi r_0\xi/\lambda z)\}$ in the plate transmittance specified in Eq. (9) is responsible of the optimum peak intensity in the discussed RVs. Such attribute of RVs can be useful in optical trapping and other applications. However, it is possible, and sometimes convenient, the use of other phase plates for generation of RVs. Such phase plates are specified by replacing the factor $sgn\{J_q(2\pi r_0\xi/\lambda z)\}$, in Eqs. (8) and (9), by a different radial phase modulation, which is denoted by $\exp[i\chi(\xi)]$. In this case Eq. (9) adopts the modified form

$$p(\xi,\phi)=\exp\left(-i\frac{k\xi^2}{2z}\right)\exp[-i\alpha(\xi,\phi)]\exp[i\chi(\xi)]\exp(iq\phi). \tag{12}$$

An example of alternate radial phase is $\chi(\xi)=2\pi r_0\xi/\lambda z$. In this case, the last two factors in the plate transmittance, Eq. (12), corresponds to the helical axicon (HA) [16]. This phase plate generates RVs with smaller peak intensities and wider bright annuli in comparison to optimal RVs. For the topological charge $q=0$, the HA becomes an axicon. The annular field obtained, at the Fourier domain of this optical element (illuminated e.g. by a Gaussian beam), can be projected to a phase SLM, where an arbitrary topological charge can be added. This procedure allows the generation of RVs of arbitrary topological charge, with an invariant annular intensity distribution [14].

The simplest specification of the radial phase function is $\chi(\xi)=0$. In this case, the parameter r_0 disappears from the plate transmittance definition and the RV radius can be only moderately controlled by means of the topological charge q. Nevertheless, the features of the RVs obtained in this case (for $q\geq 1$) are useful in special applications, e. g. vortex coronography [11,12] and super-resolution fluorescence microscopy [7].

One of the remarkable beams that belong to the class specified in Eq. (3) is the GB. In this communication, the RVs generated by this beam using our setup have been analyzed in detail. Another interesting example of beam that also belongs to the mentioned class is the flat-top field with circular support. The study of the RVs generated by this field has not been performed in the present work.

Summarizing, we have established the transmittance of a phase plate that allows the optimal focusing of a beam into an RV. We have assumed that the optimal RV is the one with the maximum possible intensity. It is not difficult to understand that other desirable features of the optimal RV, as narrow annulus and high intensity gradient, are natural consequences of maximizing the peak intensity.

As a first step we established the class of beams that can be focused into an RV. As result we obtained that the phase modulation of such beams is arbitrary but their amplitude must be only dependent on the radial coordinate.

The phase modulation of the high order Bessel beams, that appear as main factor in the transmittance of the designed phase plates, has been previously employed in the efficient although approximate generation of Bessel-Gauss beams [19–21]. The annular focal field generated by the established phase plate corresponds to the optimal physically realizable implementation of the RV, in the context of the employed setup. This result is an important complement of the well-known case of optimal beam focusing into a single spot.

We illustrated by means of numerical simulations and experimentally the generation of optimal RVs by a GB. An interesting result is that the peak intensities and widths of the RVs generated with the proposed method, show very low variation if the width of the GB is fixed and the topological charge is modified in a range of values. The radius and relative transverse width of the optimal RVs are controlled by the parameters of the phase plate transmittance and the width of the input GB. Specifically, the RV radius r_0 appears explicitly in the analytical expression for the plate transmittance. Such attributes of optimal RVs facilitate its implementation and application in conditions where the features of the RVs need to be controlled.

Acknowledgments

Authors acknowledge the financial support of CONACYT, Mexico, Mexico (245510, 322012, 367016).

References

[1] M. Born, E. Wolf, Principles of Optics, Cambridge University Press, 2003.
[2] H. He, M.E.J. Friese, N.R. Heckenberg, H. Rubinsztein-Dunlop, Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity, Phys. Rev. Lett. 75 (1995) 826–829.
[3] V. Garces-Chavez, D. McGloin, M.J. Padgett, W. Dultz, H. Schmitzer, K. Dholakia, Observation of the transfer of the local angular momentum density of a multiringed light beam to an optically trapped particle, Phys. Rev. Lett. 91 (2003) 093602.
[4] A.M. Amaral, E.L. Falcão-Filho, C.B. de Araújo, Shaping optical beams with topological charge, Opt. Lett. 38 (2013) 1579–1581.
[5] M. Levenson, S.M. Tan, G. Dai, Y. Morikawa, N. Hayashi, T. Ebihara, The Vortex Via Process: analysis and mask fabrication for contact CDs < 80 nm, Proc. SPIE 5040 (2003) 344–370.
[6] Y. Liu, D. Liu, J. Hu, Single exposure general vortex phase-shift mask for contact hole, Proc. SPIE 5567 (2004) 723–731.
[7] V. Westphal, S.W. Hell, Nanoscale resolution in the focal plane of an optical microscope, Phys. Rev. Lett. 94 (2005) 143903.
[8] H.H. Arnaut, G.A. Barbosa, Orbital and intrinsic angular momentum of single photons and entangled pairs of photons generated by parametric down-conversion, Phys. Rev. Lett. 85 (2000) 286–289.
[9] A. Mair, A. Vaziri, G. Weihs, A. Zeilinger, Entanglement of the orbital angular

momentum states of photons, Nature 412 (2001) 313–316.
[10] S.M. Franke-Arnold, M.J. Barnett, Padgett, L. Allen, Two-photon entanglement of orbital angular momentum states, Phys. Rev. A 65 (2002) 033823.
[11] D. Mawet, P. Riaud, O. Absil, J. Surdej, Annular groove phase mask coronagraph, Astrophys. J. 633 (2005) 1191–1200.
[12] J.H. Lee, G. Foo, E.G. Johnson, G.A. Swartzlander Jr., Experimental verification of an optical vortex coronagraph, Physical Rev. Lett. 97 (2006) 053901.
[13] A.S. Ostrovsky, C. Rickenstorff-Parrao, V. Arrizón, Generation of the 'perfect' optical vortex using a liquid-crystal spatial light modulator, Opt. Lett. 38 (2013) 534–536.
[14] M. Chen, M. Mazilu, Y. Arita, E.M. Wright, K. Dholakia, Dynamics of microparticles trapped in a perfect vortex beam, Opt. Lett. 38 (2013) 4919–4922.
[15] J. García-García, C. Rickenstorff-Parrao, R. Ramos-García, V. Arrizón, A. S. Ostrovsky, Simple technique for generating the perfect optical vortex, Opt. Lett. 39 (2014) 5305–5308.
[16] P. Vaity, L. Rusch, Perfect vortex beam: Fourier transformation of a Bessel beam, Opt. Lett. 40 (2015) 597–600.
[17] V. Arrizón, U. Ruiz, D. Sánchez-de-la-Llave, G. Mellado-Villaseñor, A. S. Ostrovsky, Optimum generation of annular vortices using phase diffractive optical elements, Opt. Lett. 40 (2015) 1173–1176.
[18] A. David Wunsch, Complex Variables with Applications, Addison-Wesley Publishing Company, Inc., 1994.
[19] V. Arrizón, D. Sánchez-de-la-Llave, U. Ruiz, G. Méndez, Efficient generation of an arbitrary nondiffracting Bessel beam employing its phase modulation, Opt. Lett. 34 (2009) 1456–1458.
[20] M. McLaren, J. Romero, M.J. Padgett, F.S. Roux, A. Forbes, Two-photon optics of Bessel–Gaussian modes, Phys. Rev. A 88 (2013) 033818.
[21] V. Arrizón, U. Ruiz, D. Aguirre-Olivas, D. Sánchez-de-la-Llave, A.S. Ostrovsky, Comparing efficiency and accuracy of the kinoform and the helical axicon as Bessel–Gauss beam generators, J. Opt. Soc. Am. A 31 (2014) 487–492.