Optimum generation of annular vortices using phase diffractive optical elements

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Received October 22, 2014; revised January 19, 2015; accepted February 11, 2015; posted February 18, 2015 (Doc. ID 225489); published March 17, 2015

An annular vortex of arbitrary integer topological charge q can be obtained at the Fourier domain of appropriate phase diffractive optical elements. In this context we prove that the diffractive element that generates the vortex with maximum peak intensity has the phase modulation of a propagation-invariant qth order Bessel beam. We discuss additional advantages of this phase element as annular vortex generator.

OCIS codes: (050.4865) Optical vortices; (090.1760) Computer holography; (090.1970) Diffractive optics.

http://dx.doi.org/10.1364/OL.40.001173

An optical vortex (OV) is a dark hollow in an optical field, surrounded by a bright area which possesses a certain amount of orbital angular momentum (OAM) [1–3]. In general, it is possible and desirable to generate OVs immersed in optical fields with arbitrary intensity distributions [4,5]. However, in several applications, e.g., optical trapping with OAM transference, it can be convenient to employ annular OVs. In the following text the acronym OV will represent annular OV. In general, it is advantageous to implement OVs with high efficiency methods. In addition, one may require control of the radius and the transverse intensity of an OV, while the topological charge is arbitrarily changed [6–8].

We propose a method for efficient generation of an OV with arbitrary integer topological charge q, based on a diffractive optical element (DOE), whose transmittance coincides with the phase modulation of a qth order Bessel beam (BB). This DOE, which is here referred to as Bessel beam kinoform (BBK), was previously proposed as an efficient generator of such a beam [9–11]. In this Letter, we prove that the BBK is the phase DOE that allows the generation of an OV with the maximum possible peak intensity. This optimum intensity propitiates a narrow bright annular section and a high intensity gradient in the generated OV. These features may offer advantages not only in optical trapping, but also in special applications of OVs as vortex coronography [12] and superresolution microscopy [13].

The perfect vortex is defined by its complex amplitude $B_q(\rho,\phi) = \delta(\rho - \rho_0)\exp(iq\phi)$ [6], where (ρ,ϕ) denote the radial and azimuthal coordinates, ρ_0 is the vortex radius, and q is the topological charge. From the mathematical point of view, the direct method for the exact generation of the perfect vortex is based on the Fourier transform of the unbounded qth order BB. However, this unbounded beam is impossible to obtain in practice. A physically realizable form of this optical field is the Bessel-Gauss beam (BGB) with complex amplitude

$$b_q(r,\theta) = J_q(2\pi\rho_0 r)\exp[iq\theta]\exp(-r^2/w^2), \tag{1}$$

where (r,θ) are polar coordinates and w is the waist radius of the Gaussian function. The Fourier transform of the BGB can be expressed as $B_q(\rho,\phi) = C_q(\rho)\exp(iq\phi)$, where, omitting a constant phase factor, $C_q(\rho)$ is the qth order Hankel transform of the radial factor in Eq. (1), given by

$$C_q(\rho) = 2\pi\int_0^\infty rJ_q(2\pi\rho_0 r)\exp(-r^2/w^2)J_q(2\pi\rho r)\mathrm{d}r. \tag{2}$$

The radial modulation $C_q(\rho)$ is a low-pass filtered version of the delta function $\delta(\rho - \rho_0)$ in the perfect vortex. Indeed, $C_q(\rho)$ represents an annulus of radius ρ_0, whose bright section has a transverse profile approximately given by the Fourier transform of the function $\exp(-r^2/w^2)$. The maximum intensity of $C_q(\rho)$, given by

$$I_q = |C_q(\rho_0)|^2 = 4\pi^2\left[\int_0^\infty rJ_q^2(2\pi\rho_0 r)\exp(-r^2/w^2)\mathrm{d}r\right]^2, \tag{3}$$

is obtained considering that the integrand at the right side of Eq. (2) becomes positive definite only at the radial coordinate $\rho = \rho_0$.

Next we discuss phase DOEs for the efficient generation of OVs. To evaluate the intensity profiles of these OVs, they will be normalized with respect to the peak intensity I_0 [Eq. (3)], of the OV generated with the zeroth order BGB. In general, the complex amplitude of a physically realizable OV of radius ρ_0 is given by

$$B(\rho,\phi) = F(\rho)\exp[iq\phi], \tag{4}$$

where $F(\rho)$ must show a peak intensity at $\rho = \rho_0$. For the sake of simplicity and efficiency, we propose the generation of the OV field, $B(\rho,\phi)$, by performing the Fourier transform of a phase DOE, generated by a spatial light modulator (SLM), which is illuminated by a Gaussian beam. We assume that the amplitude of the input Gaussian beam at the SLM plane is

$$g(r) = \exp(-r^2/w^2), \tag{5}$$

and that the transmittance of the phase DOE is

$$t(r,\theta) = \exp[i\beta(r)]\exp(iq\theta), \quad (6)$$

for a radial function $\beta(r)$ to be determined. Although we have assumed a Gaussian beam at the waist plane, we will note later that it is possible to use a more general Gaussian field, which includes a phase factor quadratic in r. Considering that the OV in Eq. (4) is separable in the radial and azimuthal variables, and that it must be obtained by the Fourier transform of the field

$$b(r,\theta) = g(r)t(r,\theta), \quad (7)$$

the DOE transmittance $t(r,\theta)$ [Eq. (6)] has been chosen as a separable function in r and θ. Performing the Fourier transform of the field $b(r,\theta)$ to obtain the OV complex amplitude $B(\rho,\phi)$, one obtains that the radial factor $F(\rho)$ [in Eq. (4)] is, omitting a constant phase,

$$F(\rho) = 2\pi\int_0^\infty r\,\exp[i\beta(r)]\exp(-r^2/w^2)J_q(2\pi\rho r)\mathrm{d}r. \quad (8)$$

The integral in (8) represents the qth order Hankel transform of the radial factor in $b(r,\theta)$.

It is desirable that the physically realizable OV is made up of a narrow bright annulus of radius $\rho=\rho_0$. We next establish the DOE transmittance with a radial phase $\beta(r)$ that allows the generation of the OV with the maximum possible intensity at $\rho=\rho_0$. For brevity, we will refer to this OV and the radial phase $\beta(r)$ as the optimum ones. To establish the optimum radial phase $\beta(r)$, we first employ Eq. (8) to obtain

$$|F(\rho_0)| = 2\pi\left|\int_0^\infty f_{\mathrm{pos}}(r)\exp[i\beta(r)]\mathrm{sgn}[J_q(2\pi\rho_0 r)]\mathrm{d}r\right|, \quad (9)$$

where $f_{\mathrm{pos}}(r) = r\,\exp(-r^2/w^2)|J_q(2\pi\rho_0 r)|$ is a positive definite function in the integration domain, and "sgn" denotes the sign function. Next we apply the continuous form of the triangle inequality [14] to obtain the relation

$$|F(\rho_0)| \le 2\pi\left|\int_0^\infty f_{\mathrm{pos}}(r)\mathrm{d}r\right|, \quad (10)$$

where the right side represents an upper bound value for $|F(\rho_0)|$. Finally, it is straightforward to establish from Eq. (9) that $|F(\rho_0)|$ attains this upper bound value if

$$\exp[i\beta(r)] = \mathrm{sgn}[J_q(2\pi\rho_0 r)]. \quad (11)$$

In this case, the transmittance of the phase DOE in Eq. (6) becomes

$$t(r,\theta) = \exp[iq\theta]\mathrm{sgn}[J_q(2\pi\rho_0 r)]. \quad (12)$$

So far we have proved that the maximum possible intensity of the OV (with topological charge q) at the radial coordinate $\rho=\rho_0$ is obtained, with the described method, by the BBK, whose phase transmittance is specified by Eq. (12). It is not difficult to understand that other desirable OV features, such as narrow bright annulus and high intensity gradient, can be expected as natural consequences of maximizing the OV peak intensity.

Next we evaluate the peak intensity and the full width at half-maximum (FWHM) intensity of the bright annular sections in OVs generated by BBKs. Given the analytical nature of the result proved above, it is not strictly necessary to compare the performance of BBKs with other phase DOEs. However, to illustrate the optimality of the BBK regarding the considered features, we also evaluate another phase DOE, the helical axicon (HA), which is defined by Eq. (6), with radial phase $\beta(r) = -2\pi\rho_0 r$. Both the HA and the BBK have been previously employed for the approximate generation of BGBs [11]. In the numerical simulations, we will consider values for the normalized radial coordinate ρ/ρ_0, and the waist of the Gaussian field in Eq. (5) will be given as $w = Q\rho_0^{-1}$, for different values of Q. The computed OV intensities are normalized with respect to the intensity I_0, obtained from Eq. (3) for $q=0$, which corresponds to the peak intensity of the OV generated by the BGB $b_0(r,\theta)$, defined in Eq. (1).

In Fig. 1 we illustrate the transverse intensity profiles of OVs with topological charge $q=0$ generated by BBKs [(a), (c)] and HAs [(b), (d)]. The waist radii of the considered Gaussian beams are $w=3\rho_0^{-1}$ [(a), (b)] and $w=7\rho_0^{-1}$ [(c), (d)]. As expected, BBKs generate OVs with larger peak intensities and narrower bright sections than the OVs generated by HAs with equal parameters. These features of the OVs generated by BBKs remain for different values of the parameters q and w. To justify this assertion, we computed (for a variety of cases) the intensity gain, $\max\{|F_q(\rho_0)|^2/I_0\}$, equal to the peak value of the normalized intensity profile of OVs and their FWHM values.

In Fig. 2 we show the intensity gains (left plots) and the normalized FWHMs (right plots) of OVs generated by BBKs and HAs, for different values of q and w. The normalization of FWHM is made with respect to the OV radius ρ_0. The waist radii of the Gaussian beams are $w=3\rho_0^{-1}$ [(a), (b)], $w=7\rho_0^{-1}$ [(c), (d)], and $w=10\rho_0^{-1}$

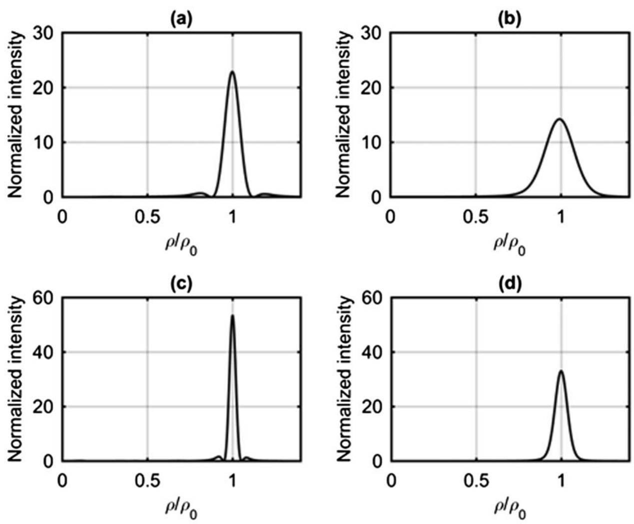

Fig. 1. Transverse intensity profiles of OVs with topological charge $q=0$ generated by (a), (c) BBKs and (b), (d) HAs. The waist w of the employed Gaussian beam is (a), (b) $3\rho_0^{-1}$ and (c), (d) $7\rho_0^{-1}$.

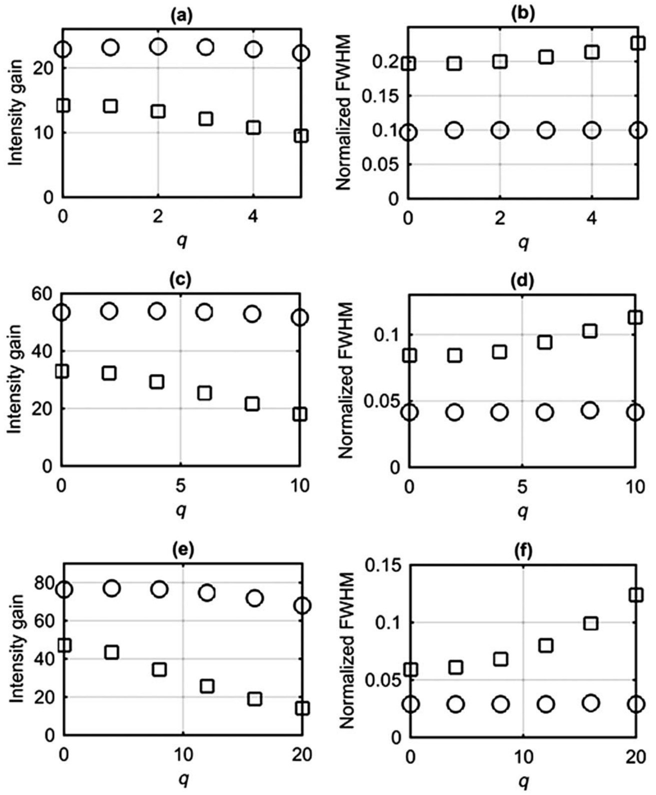

Fig. 2. Intensity gain (left) and FWHM (right) in OVs generated by BBKs (circles) and HAs (squares), for different topological charges q. The waist radius w of the Gaussian beam illuminating the DOEs is $3\rho_0^{-1}$ (top), $7\rho_0^{-1}$ (middle), and $10\rho_0^{-1}$ (bottom).

[(e), (f)]. The values of the computed parameters are marked by circles (BBK) and squares (HA). The larger intensity gains in OVs generated by BBKs are an expected result, because of the optimum character of such DOEs. On the other hand, the computed values for the FWHM confirm that the OVs generated by BBKs are narrower than the OVs generated by HAs. Besides these results, the relatively low variation in the gains of OVs generated by BBKs and the corresponding FWHMs are remarkable, for different topological charges and a fixed w. It is noted that both the OV peak intensity and the FWHM are controlled by the waist radius of the Gaussian beam used for illumination. In particular, the control of the FWHM in the OV can be crucial for different applications of this structured field.

For the experimental generation of OVs with the described approach, we employ the optical setup depicted in Fig. 3. In this setup, the light beam from a He–Ne laser source (LS) is cleaned and expanded by a spatial filter (SF), and collimated by a lens (L). The result is a Gaussian beam (GB) that illuminates a phase spatial light modulator (SLM). The phase modulation encoded in the SLM is formed by three factors. The first and second factors are the phase of the DOE under test, and a quadratic phase modulation that realizes the Fourier transform. The third factor is a linear phase modulation, whose purpose is to remove from the OV field the unmodulated fraction of the light reflected by the SLM. The intensity of the OV, obtained at the focal plane of the Fourier transforming lens encoded in the SLM, is recorded by a CCD, which is not shown in the setup.

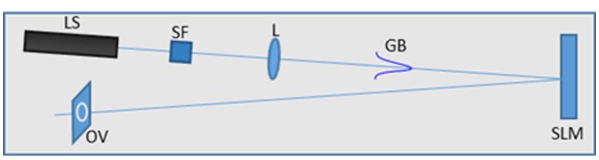

Fig. 3. Optical setup for generation of OVs at the Fourier domain of phase DOEs implemented on an SLM.

In our experiment, the waist of the generated Gaussian beam is not at the plane of the SLM. Thus, the complex amplitude of this beam presents a quadratic phase modulation (in the radial coordinate) at the SLM plane. This quadratic phase, whose curvature radius is relatively large, can be considered together with the quadratic phase encoded in the SLM. The combined effect of the two phases is equivalent to a modified Fourier transforming lens, whose focusing power is the sum of powers of the two quadratic phases.

Since the Fourier transforming lens, encoded in the SLM, is at the same plane of the function to be transformed, the OV field generated at the focal plane of such a lens, also presents a quadratic phase modulation in the radial coordinate. However, this phase modulation does not affect the intensity profile of the OV, established in the theoretical analysis.

Figure 4 shows the experimentally recorded intensities of OVs, of topological charges $q = 0$ and $q = 4$, generated by BBKs and HAs encoded into the phase SLM (Model PLUTO, Holoeye Photonics LG). The asymptotic radial period of the generated DOEs was $\rho_0^{-1} = 208\ \mu\mathrm{m}$ and the width radius w of the generated Gaussian beam was approximated to $w = 7\rho_0^{-1}$, at the SLM plane. The Fourier transforming lens encoded in the SLM, with a focal length of 50 cm, generates OVs with a diameter approximated to 3 mm.

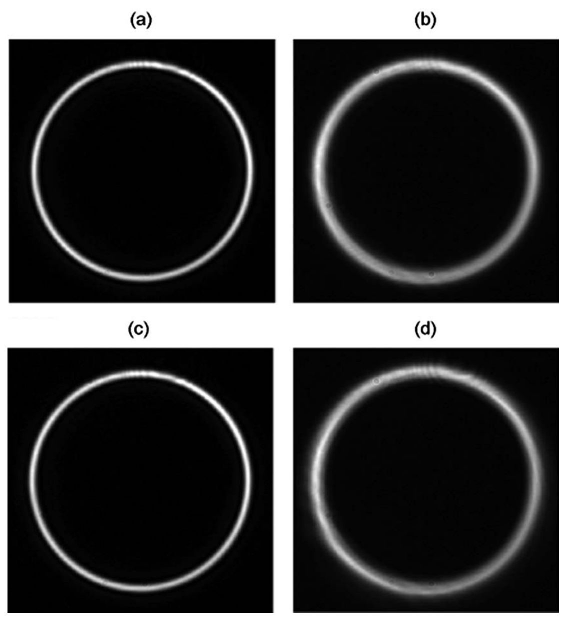

Fig. 4. Experimentally recorded intensities of OVs, of topological charges (a), (b) $q = 0$ and (c), (d) $q = 4$, generated by (a), (c) BBKs and (b), (d) HAs, which are illuminated by a Gaussian beam whose width radius was approximately $w = 7\rho_0^{-1}$.

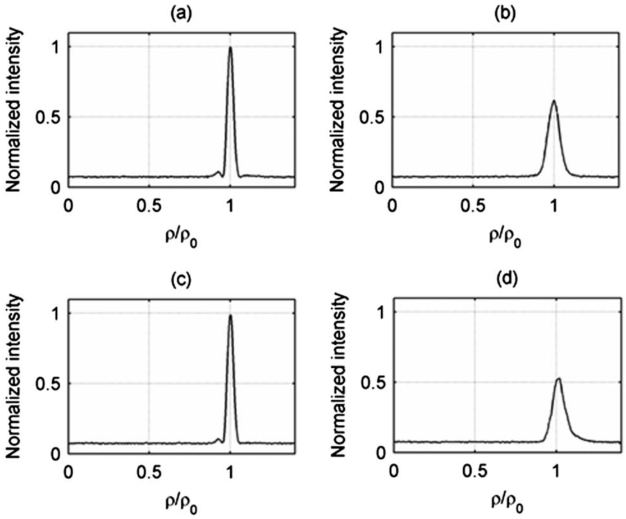

Fig. 5. Transverse intensity profiles corresponding, respectively, to the OVs displayed in Fig. 4.

Figure 5 presents the transverse intensity profiles of the OVs displayed in Fig. 4, normalized respect to the peak intensity obtained with the BBK for $q = 0$ [Fig. 5(a)]. The features of the experimental OVs (intensity profile, intensity peaks, and FWHM) are quite similar to those of the numerically generated OVs [Figs. 1(c) and 1(d)], which also correspond to $w = 7\rho_0^{-1}$. For example, the FWHM values of the experimental OV profiles in Figs. 5(a) and 5(b) are 0.045 and 0.083, respectively. These values show a close coincidence with the corresponding numerical FWHMs (0.042 and 0.084), shown in Fig. 2(d) for $q = 0$. The high similitude between the OVs in Figs. 4(a) and 4(c), and between their corresponding transverse profiles [Figs. 5(a) and 5(c)] confirms experimentally the low variation of the OVs generated by BBKs, for different topological charges and fixed w, which was previously established in the numerical simulations.

Summarizing, an OV with topological charge q and radius ρ_0 can be generated at the Fourier domain of a generic phase DOE, which is illuminated by a Gaussian beam. We proved that the phase DOE that generates the OV with the maximum possible peak intensity is the kinoform of the qth order BB, whose transmittance is $\mathrm{sgn}[J_q(2\pi\rho_0 r)]\exp(iq\theta)$.

In numerical simulations and experiments, we illustrated the optimum intensity peaks of the OVs generated by BBKs. For comparison, we also included the results obtained with HAs. We note that the optical power of the field transmitted by the generic phase DOE is equal to the optical power of the Gaussian beam used as an illumination source. Therefore the OV with the maximum peak intensity, generated by the BBK, is likely to be narrower than the OVs generated with other phase DOEs. We included the computation of the FWHM in OVs generated by BBKs and HAs, confirming such expected result. It is also concluded that the narrower OV with the maximum peak intensity presents necessarily an optimum intensity gradient, which is another desirable attribute of this field.

In Fig. 2, the relatively low variance in the intensity gain (and FWHM) of OVs generated by BBKs of different topological charges, which are illuminated by a common Gaussian beam, is remarkable. We have found that, in general, the intensity gains of OVs generated by BBKs, which are illuminated by a Gaussian beam of waist radius $w = Q\rho_0^{-1}$, present a relatively low variance for topological charges q in the range $[0, 2Q]$. The general validity of this result can be stated in the base of the analytical expression for the peak intensity $|F_q(\rho_0)|^2$. However, such an analysis is out of the scope of this Letter.

We also evaluated the OVs generated by the phase DOE with transmittance $t(r,\theta) = \exp(iq\theta)$, which corresponds to the generic transmittance in Eq. (6) with radial phase $\beta(r) = 0$. The performance of this DOE, regarding the peak intensity and width of the OVs that it generates, is poor compared with both the BBK and the HA.

U. Ruiz thanks The Conacyt, Mexico, for its financial support under Grant No. 245510.

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