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Laguerre–Gauss beams versus Bessel beams showdown: peer comparison

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We present for the first time a comparison under similar circumstances between Laguerre–Gauss beams (LGBs) and Bessel beams (BB), and show that the former can be a better option for many applications in which BBs are currently used. By solving the Laguerre–Gauss differential equation in the asymptotic limit of a large radial index, we find the parameters to perform a peer comparison, showing that LGBs can propagate quasi-nondiffracting beams within the same region of space where the corresponding BBs do. We also demonstrate that LGBs, which have the property of self-healing, are more robust in the sense that they can propagate further than BBs under similar initial conditions.

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The last two decades have seen a need to find new ways to increase the channels for communications. For that purpose, researchers have turned their attention to electromagnetic structured wave fields, particularly those carrying orbital angular momentum. Recent studies have shown that these structured wave fields can boost classical and quantum communications, and are an excellent option to improve quantum entanglement and quantum cryptography [1–5].

For many years, Gaussian beams were the preferred wave fields, and researchers investigated their use in communications and many other applications [6]. However, this only lasted until the arrival of the families of the nondiffracting structured Bessel and Bessel–Gauss beams (BGBs) [7,8].

When Bessel beams (BBs) were introduced and demonstrated, the fundamental Gaussian beam was used as a reference in order to evaluate the properties of the BBs. It was observed that under same initial total power and the same initial full width at half-maximum in spot size, they could present advantages on propagation and power-transport efficiency over Gaussian beams, and in some other instances, they could be comparable [7–10].

From then on, there have been a great number of studies comparing the fundamental bell-shaped Gaussian beam and the multi-ringed BB under a variety of conditions in free space, including the generation of nonlinear effects and in the accelerators of particle beams. In some cases, it has been found that one beam may have better performance over the other or vice versa, depending on the physical situation [11–21].

In a radially symmetric laser system, the Gaussian beam is the lowest-order mode of an infinite family of structured multiringed modes, the standard Laguerre–Gauss beams (LGBs) [22,23]. There are also the elegant LGBs, which, like the BGBs, have a complex argument [8,24]. These beams have also been subjects of comparison, and it was found that, for a given set of parameters, both propagate similarly [25]. These kinds of beams are structured only within the finite, conical volume where they can exist. Beyond this volume, both modify their structure into a ring in the far field [26,27].

By reviewing the literature, one finds that high-order, standard LGBs with orbital angular momentum have received little attention. This means that their whole set of properties have not been fully studied to be able to properly make use of them. Some of their properties can be fundamental for a variety of potential applications, such as those mentioned above.

The purpose of this Letter is to present for the first time a proper comparison of BBs with their peer family of radially symmetric Gaussian beams, the high-order LGBs. We demonstrate that by imposing the right conditions, the LGBs propagate quasi-nondiffracting beams like BBs within the same conical volume of existence of BBs. Additionally, since they are structurally stable outside that volume (although they do diffract), they propagate further than BBs, which are restricted to propagate over finite distances. Our investigations reveal that LGBs also possess a more robust property of self-healing, showing better recovery than BBs.

We start by defining LGBs, BBs, and BGBs, but the showdown will be mainly between the LGBs and the BBs. LGBs are solutions of the paraxial wave equation with the cylindrical coordinates (r, φ, z) and can be expressed as [6]

$$U_{\rm LG}(r,\varphi,z) = A_0 \frac{w_0}{w(z)} \left(\frac{\sqrt{2}r}{w(z)}\right)^{|m|} L_n^{|m|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left[\frac{-r^2}{w^2(z)} - \frac{ikr^2}{2R(z)} + i(2n+|m|+1)\Phi(z) - im\varphi\right], \quad (1)$$

where A_0 is a constant which stands for the amplitude, $L_n^{|m|}$ are the associated Laguerre polynomials, with n and m as the radial and azimuthal orders, respectively, w_0 is the beam waist of the beam width $w^2(z) = w_0^2[1 + (z/L_D)^2]$, $R(z) = z[1 + (L_D/z)^2]$, and $\Phi(z) = \tan^{-1}(z/L_D)$, where $L_D = kw_0^2/2$ and $k = 2\pi/\lambda$.

On the other hand, the initial conditions at $z = 0$ for the BBs and BGBs can be expressed as [7,8]

$$U_B(r,\varphi) = A_B J_m(k_t r)\exp(-im\varphi), \quad (2)$$

$$U_{\rm BG}(r,\varphi) = A_{\rm BG} J_m(k_t r)\exp\left(\frac{-r^2}{w_{\rm BG}^2}\right)\exp(-im\varphi), \quad (3)$$

where $J_m(\cdot)$ is the first-kind Bessel function of order m, k_t is the transverse component of the cone of the wave vectors of the BBs and BGBs [7,8], A_B, $A_{\rm BG}$ are the respective amplitudes of the beams, and $w_{\rm BG}$ the waist of the BGB.

In order to obtain the parameters of the LGB profile in Eq. (1) that makes its profile similar to the corresponding BB in Eq. (2), we follow the procedure described in [28]: rewrite the Laguerre–Gauss differential equation as an inhomogeneous Bessel differential equation, and look for the asymptotic behavior when $n \gg 1$. After some algebra, we get

$$e^{-r^2/w^2(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|m|} L_n^{|m|}\left(\frac{2r^2}{w^2(z)}\right) \approx \frac{\Gamma(n+|m|+1)}{n!N^{|m|/2}} J_m\left(2\sqrt{2N}\frac{r}{w(z)}\right), \quad (4)$$

where $N = n + (|m| + 1)/2$ and $\Gamma(\cdot)$ is the gamma function. This equation determines the relation among the main LGB parameters in Eq. (1), and the waist, radial, and azimuthal indexes n and m with the radial frequency k_t of the BB in Eqs. (2) and (3). This is the core of our investigation, as *it establishes the requirement for an LGB to behave similarly to a BB.* Setting this value at $z = 0$, we have the relation $k_t = 2\sqrt{2N}/w_0$.

It is easy to see that for any pair of indexes m and n, the rings in a high-order LGB are confined within a disk of radius $R = w_0\sqrt{2N}$, see, e.g., [29]. Thus, for the showdown we use this radius to define the extent of the BB and the BGB waist, i.e., $w_{\rm BG} = R$. A typical example of the intensity patterns of LGBs and BBs at $z = 0$ is presented in Fig. 1. The left column is for the pair of indexes $m = 0$ and $n = 10$, while the right column is for $m = 5$ and $n = 10$. The bottom row shows the comparison of the corresponding intensity profiles of the beams, including that of the BGB. The intensity profiles of the three beams in the central region are practically the same [see Figs. 1(e) and 1(f)]. The outermost rings differ in amplitude and frequency due to the nonuniform distribution of the zeros and the polynomial growth of the Laguerre–Gauss

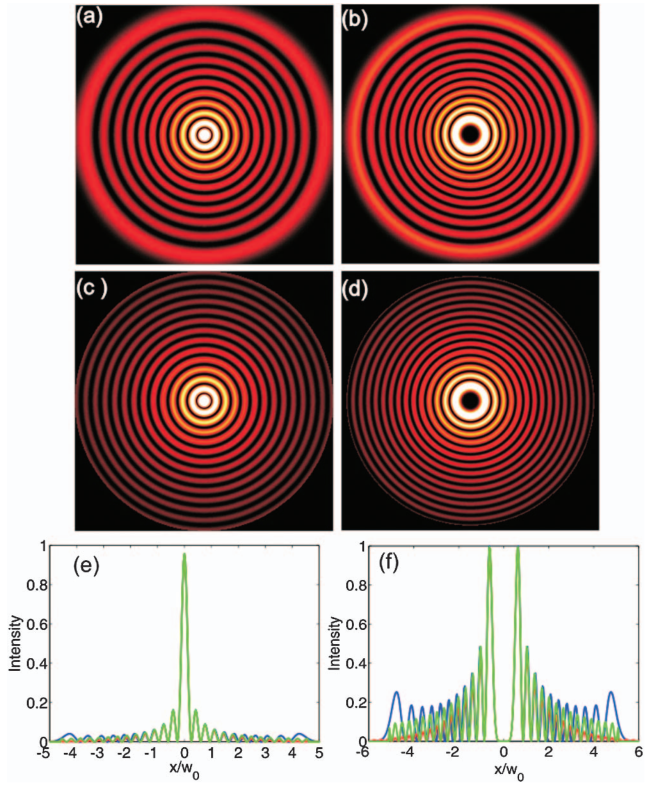

Fig. 1. Intensity patterns and profiles for Laguerre–Gauss beam (LGBs) and Bessel beams (BBs). Left column: $m = 0$, $n = 10$; right column: $m = 5$, $n = 10$. Top row: LGB; middle row: BB; and bottom row: intensity profiles, LGB (blue line), BB (green line), and BGB (red line).

function. Studying the transverse profiles of the beams by varying the values of the m and n, and by making m any positive integer and making $n \geq 5$, we observed that the likeness between the LGBs and BBs occurs within a disk of radius of $2w_0$ that encloses approximately the first $n/2$ rings.

Having determined the radial frequency for the BBs and the BGBs and their transverse extent, we can now obtain their maximum propagation distance, which is $Z_{\max} = R/\tan\theta \approx R/(k_t/k) \equiv L_D$. *This relation establishes that under similar circumstances, the maximum propagation distance of a BB is practically identical to the diffraction length of a LGB. This is a very remarkable relation in our analysis, since the distance L_D defines the space within which the LGB can be considered quasi-collimated and their ringed structure does not change significantly when propagating with minimum diffraction, like a quasi-nondiffracting BB.*

To demonstrate this, we next investigate the propagation of the field distributions in Fig. 1 by numerically solving the paraxial wave equation $2ik\partial U/\partial z + \nabla^2 U = 0$ for the BB and comparing it with the closed analytical solution of the LGB in Eq. (1). The transverse radial coordinate is normalized to the beam's waist w_0, and the propagation distance z with respect to L_D. In Fig. 2, the behavior of the intensity in the plane x–z for the LGB (top) and BB (bottom) for $m = 0$, $n = 10$ is shown. For reference, in both images, the triangular region of existence of the BB and the external hyperbola of the LGB have

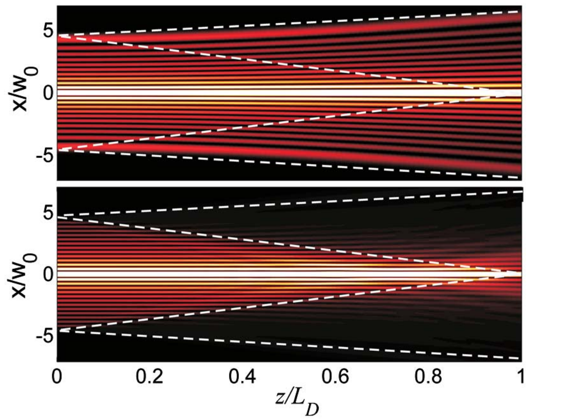

Fig. 2. Behavior of the propagation on the plane x–z for a LGB (top) and a BB (bottom) with $m = 0$ and $n = 10$. The conical region for BB and the size of the LGB are indicated with a white dashed line.

been marked with white dashed lines [29,30]. A closer look will show that the behavior of both beams is practically identical within the region of existence of the BB. Outside this region, the BB has transformed into an outgoing Hankel wave [30,31] while the LGB maintains its ringed structure but is clearly spreading.

The on-axis intensity for the three beams when $m = 0$ and $n = 10$ are shown in Fig. 3. At $z = L_D$, the normalized on-axis intensities of the BB and BGB decay to less than 0.25, while that of the LGB decays to 0.5. However, for propagation distances smaller than $0.9L_D$, the on-axis intensity of the BB is higher than that of the LGB.

In Fig. 4, the propagation of beams with orbital angular momentum with $m = 5$ is presented. As expected, we observe that the dynamics are the same as for the case when $m = 0$.

The above result based on Eq. (4) is very relevant in a peer comparison between LGBs and BGBs, since the control of the LGB indexes has been seen as a drawback for the optical manipulation and detection of Laguerre–Gauss modes in studies of quantum entanglement [3,4]. Our results show that this should not be the case for multi-ringed, high-order LGBs, thus opening the possibility for investigating new potential applications.

Finally, we present another feature of the multi-ringed LGB that is believed to be characteristic of nondiffracting BBs:

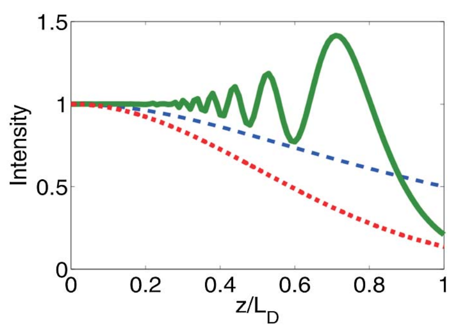

Fig. 3. On-axis intensity for one diffraction distance of the BB (solid green line), BGB (dotted red line), and LGB (dashed blue line) for the case in which $m = 0$ and $n = 10$.

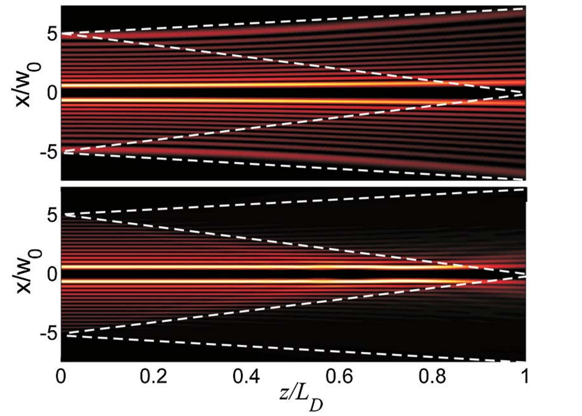

Fig. 4. Same as Fig. 2, but with $m = 5$ and $n = 10$.

self-healing [32,33]. Such self-healing has been reported for LGBs with one and two rings [34,35]. In Fig. 5, we present the results at different propagation stages obtained from a multi-ringed LGB and a BB that was initially obstructed by an opaque disk placed off-axis. We can observe that the dynamics of the LGB (left) are very similar to that of the peer BB (right). But, since the LGB can propagate further than the BB, it displays more robustness and has a better recovery. In other words, LGBs can carry the required information more efficiently beyond the distance than BBs can.

In conclusion, we have demonstrated that when compared under similar circumstances, Laguerre–Gauss beams (LGBs) can be a better option than Bessel beams (BBs) for many applications that are currently regarded as being proper for BBs.

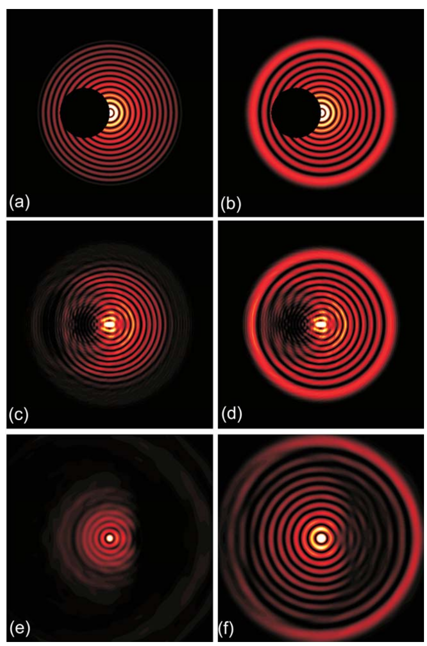

Fig. 5. Self-healing of (a) BBs and (b) LGBs with $m = 0$ and $n = 10$ propagated at $z = 0$. (c) BBs and (d) LGBs at $z = 0.2$. (e) BBs and (f) LGBs at $z = L_D$. First column BB, second column LGB. Both columns have an identical density scale. Notice that at $z = L_D$, the BB has almost disappeared, while the LGB is clearly self-healing.

We obtained the parameters to perform a peer comparison between an LGB and a BB, and demonstrated that in the region of space where a BBs propagates as quasi-nondiffracting, its peer LGB does as well. We demonstrated that LGBs possess a more robust property of self-healing, and have the capability of propagating over longer distances than their peer BBs. As LGBs are being generated very efficiently in recent years, they can be an excellent option for present and future applications [36–38].

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