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Composite modified Luneburg model of human eye lens

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A new lens model based on the gradient-index Luneburg lens and composed of two oblate half spheroids of different curvatures is presented. The spherically symmetric Luneburg lens is modified to create continuous isoindicial contours and to incorporate curvatures that are similar to those found in a human lens. The imaging capabilities of the model and the changes in the gradient index profile are tested for five object distances, for a fixed geometry and for a fixed image distance. The central refractive index decreases with decreasing object distance. This indicates that in order to focus at the same image distance as is required in the eye, a decrease in refractive power is needed for rays from closer objects that meet the lens surface at steeper angles compared to rays from more distant objects. This ensures a highly focused image with no spherical aberration.

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The eye lens, by virtue of its biological nature and protein distributions, is an optically inhomogeneous lens with a gradient refractive index (GRIN). A number of GRIN models have been proposed in the past, the most famous of which is Maxwell's fish-eye [1]. This lens, however, is not found in any animal eye and is limited in practical applications [2]. The Luneburg lens [3] with a wider range of applications has a simpler mathematical description and has been proposed as a model for the eye lens [4,5]. However, the spherical symmetry of the Luneburg lens [4] is inapplicable to the human lens, which approximates an oblate spheroid.

The eye lens is the dynamic component of the ocular system that changes or accommodates its shape to correctly focus the eye in response to visual demand. As the lens shape alters, there will be an internal cellular redistribution that will cause a change in the form of the GRIN profile [2]. Hence, for any accommodating lens, there cannot be a single mathematical expression to describe the GRIN. The original Luneburg lens is a sphere with a GRIN that is free from spherical aberrations [3,5]. The form of the GRIN changes for every pair of object and image conjugate points and in this way is similar to the eye lens [4].

It is possible to geometrically transform the original spherical Luneburg lens into a symmetrical oblate spheroid while maintaining its aberration free property [6,7]. However, the human lens is not symmetrical and, taking this into consideration, alternative GRIN models with asymmetric bi-spheroidal shapes have been proposed [8,9]. In addition to asymmetries and minimizing aberrations, in order to have a realistic and applicable model of the eye lens, its accommodating ability needs to be taken into account so that image quality can be maintained for a wide range of object distances.

In this Letter, we present a dynamic model of the human lens constructed with a shape akin to that found in the eye using two separate spheroidal hemispheres. The isoindicial contours and their first-order derivatives are assumed to be continuous in the equatorial plane. This condition creates a match in the GRINs of both hemispheres to prevent any discontinuity of the ray paths. The model allows for variable curvatures for the anterior and posterior sections, but for the purposes of this Letter, the geometry is fixed. We investigate the image formation capacity of such a lens for distant sources comparing it with existing models. We show that, based on the Luneburg lens theory, the model requires modification of its GRIN distribution for each distance in order to produce the appropriate image. This model will be referred to as the Composite Modified Luneburg (CML) lens.

From Luneburg theory [3], the rays traverse an inhomogeneous medium of spherical symmetry and the ray paths are described by the Eikonal equation which is given in terms of the polar and radial coordinates as

$$\left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\psi}{\partial\theta}\right)^2 = n^2(r), \tag{1}$$

where ψ describes the optical path of the rays and $n(r)$ the refractive index of the lens. The surfaces $\psi =$ constant are associated with the optical wavefronts and Eq. (1) allows for nonparaxial rays. Since the refractive index depends only on the radial coordinates, this equation can be solved using separation of variables. For an arbitrary ray in an inhomogeneous medium the solution is given by

$$\psi = K\theta \pm \int_{r_0}^{r} \sqrt{n^2 - \frac{K^2}{r^2}}\mathrm{d}r, \tag{2}$$

where K is the separation constant and is constant for any particular ray [5,10,11]. Rearranging this equation and applying the theorem of Jacobi gives the following:

$$\theta - \theta_i = \pm K \int_{r_0}^{r} \frac{\mathrm{d}r}{r\sqrt{n^2(r)r^2 - K^2}}, \tag{3}$$

where (r_0, θ_i) are the coordinates of the point of ray entry into the lens as shown in Fig. 1. Calculating the derivative of Eq. (3) with respect to the radial coordinate yields

$$K = n(r)r \sin\varphi, \tag{4}$$

where $\sin\varphi = rd\theta/dr[1 + (d\theta/dr)^2r^2]^{-1/2}$ and the angle φ is formed between the radial vector and the tangent to the ray path (Fig. 1) [3]. This expression is known as the generalized Snell's law for inhomogeneous media and represents a family of rays, each determined by the particular value of K. This value can be obtained at the point of incidence of the Luneburg lens, where $K = n(r_0)r_0 \sin\alpha_0 = n_0 r_0 \sin\alpha_0$, with n_0 the external refractive index and α_0 the angle that the ray makes with the optic axis (Fig. 1).

By normalizing the system such that the radius is a unit radius and the external refractive index $=1$, then $0 \le K \le 1$. After further calculations [3,5] Eq. (3) becomes

$$\int_{r^*(K)}^{1} \frac{K\mathrm{d}r}{r\sqrt{n^2(r)r^2 - K^2}} = \frac{1}{2}\left(\sin^{-1}\frac{K}{r_0} + \sin^{-1}\frac{K}{r_1} + 2\cos^{-1}K\right), \tag{5}$$

where r^* is the minimum value of the radial position for each ray determined by K as shown in Fig. 1. This equation indicates that the only unknown quantity is the refractive index. For any given pair of conjugate points and for each ray that joins them, the refractive index has to be obtained from the integral Eq. (5). Without loss of generality, the position of the image point can be fixed, so that only a change in position of the object point will result in a change in the GRIN distribution. Such a change can be compared to the process of accommodation in the human lens [2] and may serve as a suitable model for the biological process. The change in the anterior and posterior curvatures of the eye lens with accommodation is also considered.

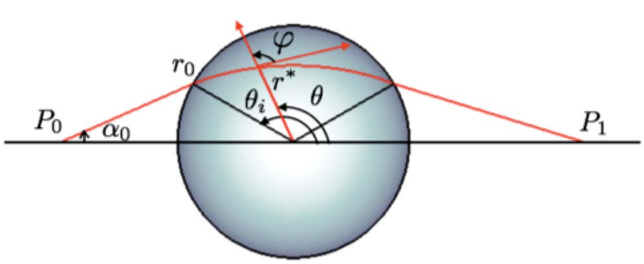

Fig. 1. Luneburg lens showing the geometric parameters of the ray path.

The index in the spherical Luneburg lens is described by a function $n(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$ and x, y and z are Cartesian coordinates. Setting $\rho = \sqrt{x^2 + y^2}$ and assuming symmetry around the z axis gives two forms of the equation that can be used to obtain the desired spheroidal shape:

$$\left(\frac{1}{s}y\right)^2 + z^2 = r^2 \quad \text{or} \quad y^2 + \left(\frac{1}{s}z\right)^2 = r^2. \tag{6}$$

In these equations, s is a constant, the value of which can compress or elongate the sphere along the y or z axis. To model the eye lens, this constant will be determined by the fusion of two spheroidal hemispheres of different curvature but the same equatorial diameter [6–9].

The model is constructed by imposing the condition that the GRINs and the axial derivatives of the anterior and posterior hemispheres match at every point on the equatorial plane. For this purpose, the refractive indices, $n_a(r)$ for the anterior and $n_p(r)$ for the posterior hemisphere, must be of the form

$$n(y,z) = \begin{cases} n_a(y,z) = \sqrt{n_c^2 - \left[\left(\frac{y}{R}\right)^2 + \left(\frac{1}{Rs_a}z\right)^2\right]} & z \le 0, \\ n_p(y,z) = \sqrt{n_c^2 - \left[\left(\frac{y}{R}\right)^2 + \left(\frac{1}{Rs_p}z\right)^2\right]} & z \ge 0, \end{cases} \tag{7}$$

where n_c is the refractive index at the center of the lens, and R is the radius of the lens measured on the equatorial plane. The scaling factor is $s_{a,p} = z_{a,p}/R\sqrt{n_c^2 - n_s^2}$ where z_a is the anterior vertex and z_p the posterior vertex. For $n_a(r)$, the variable z' is negative, and for $n_p(r)$, the variable z' is positive. Figure 2 shows the continuous, bi-elliptical isoindicial contours in the sagittal plane.

The model uses shape parameters reported in the literature [12], and the refractive index ranges from $n_c = 1.4181 \pm 0.075$ in the core to $n_s = 1.3709 \pm 0.0039$ at the edge of the lens [2]. The age dependence of the geometric parameters is estimated by the following equations [12]:

$$\begin{aligned} R &= [0.0138(\pm 0.002) * \mathrm{Age} + 8.7]/2, \\ z_p &= 0.0074(\pm 0.002) * \mathrm{Age} + 2.33, \\ z_a &= 0.0049(\pm 0.001) * \mathrm{Age} + 1.65. \end{aligned} \tag{8}$$

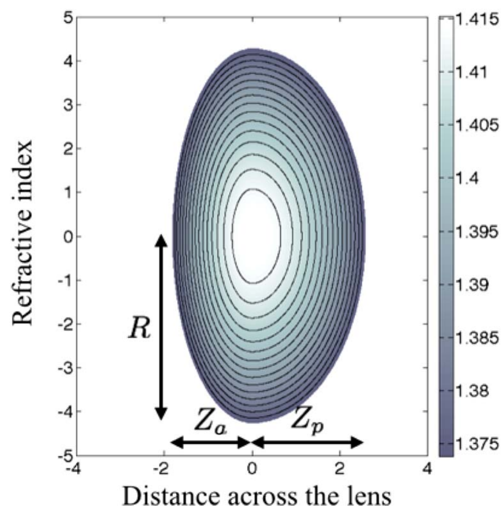

Fig. 2. Geometry of the bi-spherical model showing the continuity of the isoindicial lines at the equatorial plane.

For a lens aged 35 years, the corresponding parameters are $R = 4.4005$ mm, $z_a = 1.8215$ mm, and $z_p = 2.5890$. The curvatures of the anterior and posterior hemispheres are such that the ratio between z_a and z_p is 0.7035.

In order to investigate the imaging properties of this model, we follow a procedure similar to that described in Ref. [6] modified accordingly to fit the new geometry and the required continuity conditions. The surround refractive index is 1.336, and the surface refractive index n_s is constant. The first case investigated simulates a modification of the Luneburg model using a constant ratio z_a/z_p applicable to a 35-year-old lens [13]. In these simulations, the only parameter of the lens that can be modified is the central refractive index n_c, the parameters n_s and z_a/z_p being fixed. Changes in the GRIN are investigated for a fixed image distance of 63.05 mm from the posterior surface of the lens and for five distances from the anterior surface: infinity, 200, 100, 50, and 25 cm.

Figure 3 shows the gradient index profiles for the five object distances. It can be seen that there is a decrease in n_c as the object point moves closer to the anterior surface of the lens.

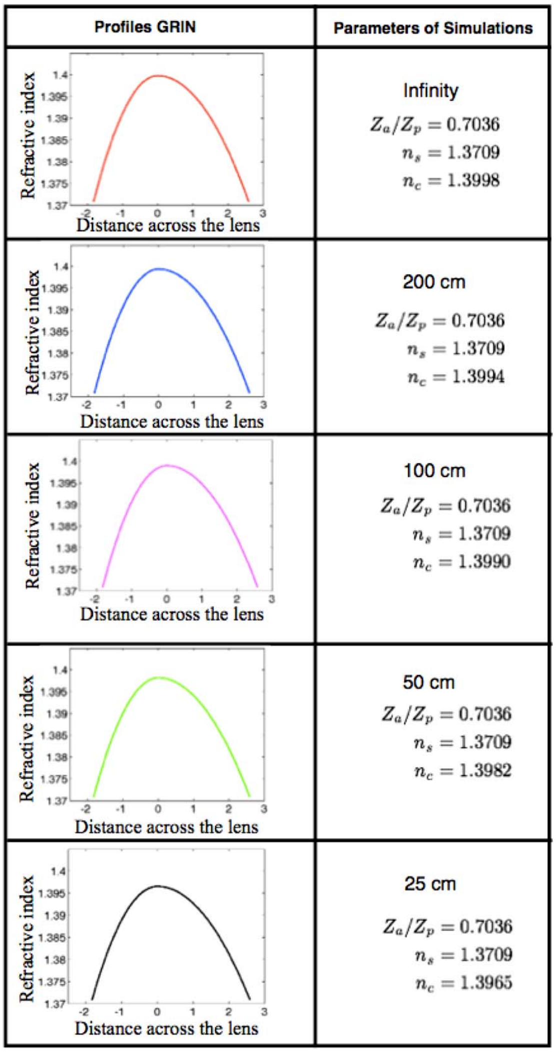

Fig. 3. Gradient refractive index (GRIN) profiles calculated for a point object at different distances. The lens has the same ratio z_a/z_p and surround refractive index $=1.336$. There is a reduction of n_c as the object gets closer to the lens.

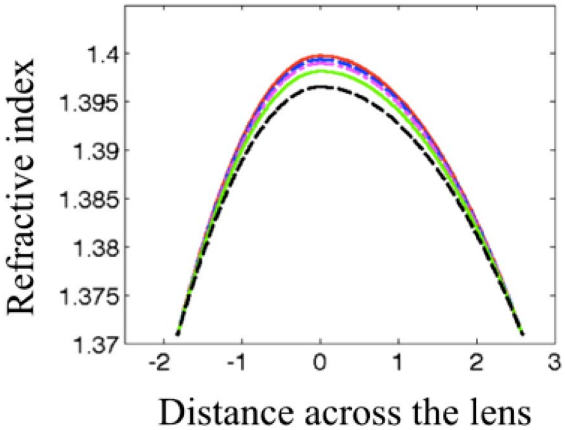

Fig. 4. Direct comparison of the GRIN profiles calculated for the different distances of the point object in Fig. 3. The lens has a fixed ratio z_a/z_p.

The variation in n_c is 0.0033 between the case of an object at infinity (for which n_c, is 1.3998) to that of an object at 25 cm (for which n_c, is 1.3965). For a direct comparison, the GRIN profiles corresponding to the five object distances are plotted in Fig. 4.

A lower n_c gives a lens with lower refractive power. It seems counterintuitive to have a lower refractive power for objects close to the lens compared to objects further from the lens. In the human lens, focusing on near objects requires the lens to increase its refractive power by increasing its curvature. This action causes an expansion of the nuclear part of the lens but does not alter the magnitude of its refractive index [14,15], which is relatively constant over the nuclear region [2,13]. This contrasts with the decrease in n_c for shorter object distances that applies to the CML lens. Unlike the biological lens that alters its shape with accommodation, this lens has a fixed geometry, and n_c decreases because the angle α_0 is more steeply inclined the closer the object is to the lens (Fig. 1). Hence, if the value of n_c remained the same as for more distant objects, the rays would refract more strongly and focus closer than the set image distance of 63.05 mm.

In conclusion, we have introduced a new dynamic model of the human lens, the Composite Modified Luneburg (CML) lens. It is constructed with two Luneburg oblate spheroidal hemispheres of different elliptical parameters but with continuous isoindicial contours and continuous first-order derivatives at the equatorial plane. Maintaining the same geometry, we investigated the changes in gradient refractive index (GRIN) that occur when an object placed at different distances is imaged onto a fixed plane simulating the retina. Our results show that there is a redistribution of the GRIN with a central refractive index that is reduced in order to focus the object as it gets closer to the lens and indicates that for an accommodating lens, there cannot be a single mathematical expression to describe the GRIN.

Our model allows for changes in the lens shape and can be used to investigate the effect of the GRIN on aberrations of the lens. Using such a variety of lens models, it is possible to create a schematic eye with a more-realistic GRIN lens and for different shapes and ages to evaluate the image quality, extent, and types of aberrations and the effects of corneal topography.

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