Splitting after collision of high-order bright spatial solitons in Kerr media

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Abstract: By numerically studying the collision between (1 + 1)-Dimensional high order bright spatial solitons in a Kerr nonlinear media we show that after the collision, the high order solitons split into a number of first order solitons that corresponds to its order. Two different collision scenarios are considered: collision between two independent high order solitons and a collision with a virtual soliton simulated by the reflection at an angle of a high order soliton at a linear interface. The results demonstrate that in both cases the high order solitons split showing minor differences.

OCIS codes: (190.0190) Nonlinear optics; (190.6135) Spatial solitons.

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1. Introduction

High order solitons have received much attention in recent years due to their application in the generation of ultra-short optical pulses and supercontinuum [1,2]. First order solitons, also called fundamental spatial solitons, are beams that propagate without spreading in a nonlinear media with an intensity dependent refractive index [3]. Different physical mechanism can lead to the generation of spatial solitons in nonlinear media [4]. The interaction between spatial solitons has been studied numerically and experimentally for the coherent [5–8] and incoherent case [9–11]. Collision between spatial solutions has been restricted for the case between fundamental ones with the same dimensionality [12–16] or different dimensionality [17].

High order solitons are bound states of overlapping fundamental solitons with different amplitudes. On propagation they present periodic behavior, the same period being independent of the order of the solitons. Different mechanisms have been proposed in order to split such overlapped solitons such as: high order linear and nonlinear effects, self-steeping, high order dispersion, and others [18–23]. The splitting of a high order soliton induced by a perturbing pulse was analyzed in where an overlapping between the beams was considered [24]. Stabilization and destabilization mechanisms after splitting for second order soliton were presented in [25].

In this paper we show that the collision of high order spatial solitons in a Kerr nonlinear medium induces separation into their first order soliton components. No initial overlapping between the solitons is considered. Two collision schemes are investigated: two high order solitons approaching at a given velocity and the collision of a soliton with its virtual image created by total internal reflection at a linear interface. The splitting of the high order soliton is obtained under both conditions the collision and reflection at an interface. The results demonstrate a very simple way to split a high order soliton into its fundamental soliton components.

2. High order bright spatial solitons

The mathematical description of a spatial soliton in a Kerr media is given by the normalized Schrödinger equation (NLSE):

$$i\frac{\partial q}{\partial Z}=\frac{1}{4}\frac{\partial^2 q}{\partial X^2}\pm\frac{L_D}{L_{NL}}|q|^2 q, \tag{1}$$

where q is the normalized amplitude of the field to the maximum intensity $I_m^{1/2}$, $L_D = n_0k_0x_0^2/2$ is the diffraction length with n_0 the linear refractive index for beam, $k_0 = 2\pi/\lambda$ with λ the wavelength, x_0 is the initial beam width, $L_{NL} = (n_2k_0I_m)^{-1}$, n_2 the nonlinear refractive index, $X = x/x_0$ and $Z = z/L_D$. The positive sign is used for a positive n_2 and the negative for the opposite case n_2<0. For a positive Kerr media, when L_D/L_{NL} = 1, the NLSE has a solution

$$q(X,Z) = \kappa \sec h(\kappa X)\exp(-\kappa^2 Z/2), \tag{2}$$

where κ is the soliton amplitude. This solution is known as the first order bright spatial soliton, or fundamental soliton. When $N = L_D/L_{NL}$ is an integer larger than 1, the solution represents N fundamental solitons that interact nonlinearly in an attractive effective potential [5]. In our case we are interested in initial conditions of the form:

$$q(X, Z=0) = N \sec h(X), \tag{3}$$

where N>1 and can be any positive integer. This solution is also known as a high order soliton that on propagation presents a periodic behavior. The period length where the initial distribution is recovered is $Z = m\pi/2$, where m is an integer. In Fig. 1 we present the numerical propagation of a soliton of fourth and fifth order in a period length. It is interesting to note that the behavior of a propagating soliton of high order presents the transverse profiles very similar to those obtained for solitons of lower order. To show this fact the intensity profiles at $Z = \pi/4$ for spatial solitons of second, third, fourth and fifth order are shown in the top row of Fig. 2. In the bottom row of Fig. 2 are shown the intensity profiles of a sixth order soliton at different propagation distances. The sixth order soliton at $Z = \pi/24$ is initially narrowed, (as the second order soliton at $Z = \pi/4$), then at $Z = \pi/18$ it is split into two (as the third order soliton at $Z = \pi/4$), at $Z = \pi/12$ it splits into three (as a forth order soliton at $Z = \pi/4$) and at $Z = \pi/8$ it splits into four beams (as the fifth order soliton at $Z = \pi/4$). It is clear that the sixth order soliton exhibits a similar dynamics as that obtained for the previous order solitons

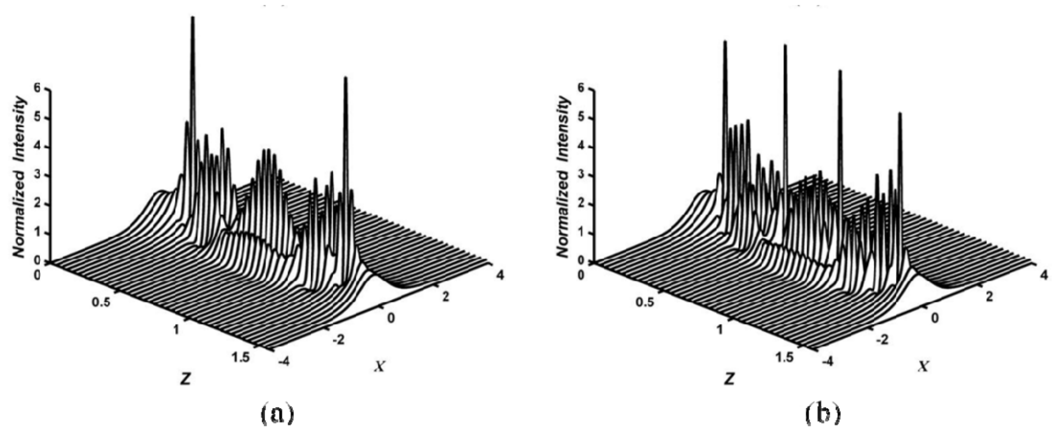

Fig. 1. Propagation of high order solitons for a distance of $Z = \pi/2$. Order of the soliton of: (a) 4th, and (b) 5th.

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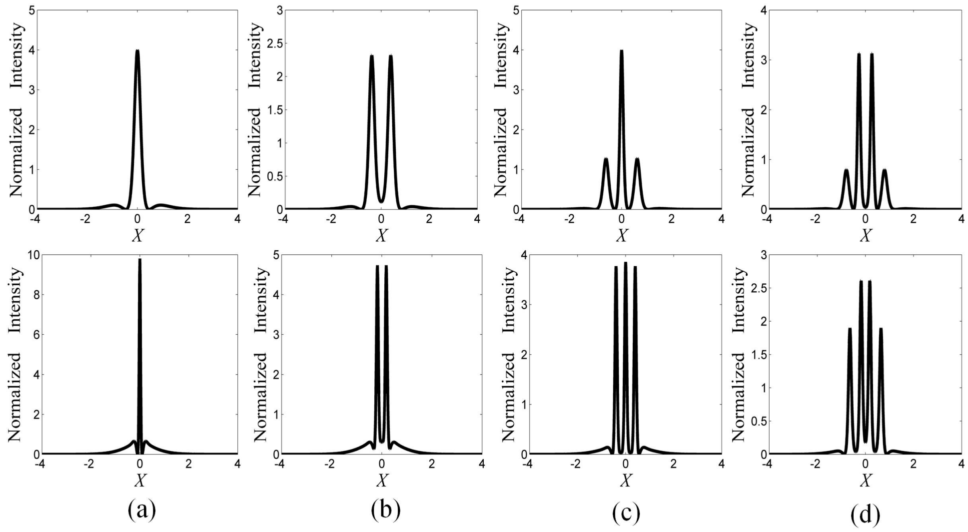

Fig. 2. Intensity profiles for different soliton orders and distances. Top row for a propagation distance of $\pi/4$ and soliton order of: (a) 2nd, (b) 3th, (c) 4th and (d) 5th. Bottom row for a sixth order soliton and propagation distance of: (a) $Z = \pi/24$, (b) $Z = \pi/18$, (c) $Z = \pi/12$ and (d) $Z = \pi/8$.

3. Symmetric collision of same order bright spatial solitons

Initially the symmetric collision of high order solitons was considered. Both solitons made the same angle with respect to the z axis. This angle defines the transversal velocity V of the soliton ($2V = \tan\theta$). The solitons were set at the same distance from the z-axis. Separation distances between the solitons larger than 2 were used in this work in order to ensure that each beam does not overlap or is affected by the other beam. Interaction between initially overlapped solitons has been considered in other works; see as example [26]. The initial condition that was used is of the form:

$$q(X, Z=0) = N \operatorname{sech}(X - X_C)\exp(-iVX) + N \operatorname{sech}(X + X_C)\exp(iVX), \qquad (4)$$

where X_c is the half of the separation distance, d, between the solitons.

After the collision both high order solitons split. The number of beams that appeared after the collision depended on the soliton order, see Fig. 3, these beams propagated in parallel way making the same angle that the incident solitons, see Fig. 4. The beams with higher intensity were closer to the propagation axis and the intensity of the remaining beams decays as the distance increased from the center. The width of the generated solitons, after collision, increased as the intensity decreased. Not all of the generated solitons are fundamentals, some of them exhibit oscillatory behavior after the collision, see Fig. 4. No substantial qualitative differences were observed for different transversal velocities (angles) and separations between the solitons.

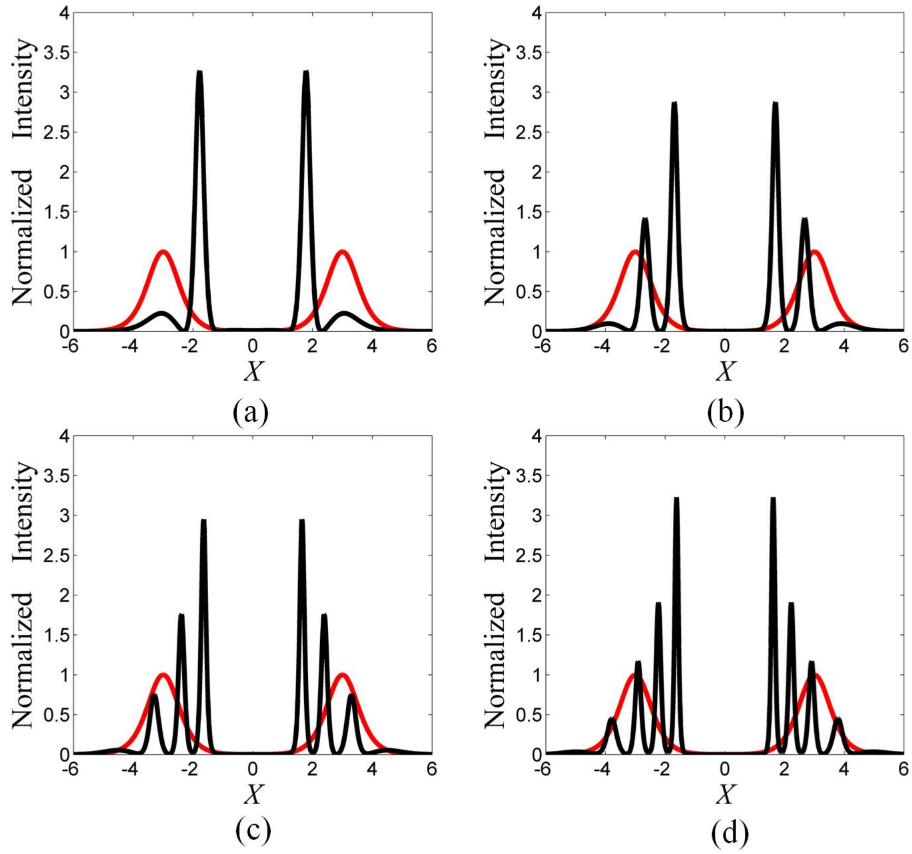

Fig. 3. Intensity profiles at $Z = 0$ (red) and $Z = 5\pi/2$ (black) for the symmetric collision of high order solitons. The initial separation was 6 and the magnitude of the transversal velocity $V = 1$. Soliton order of: (a) 2nd, (b) 3th, (c) 4th and (d) 5th.

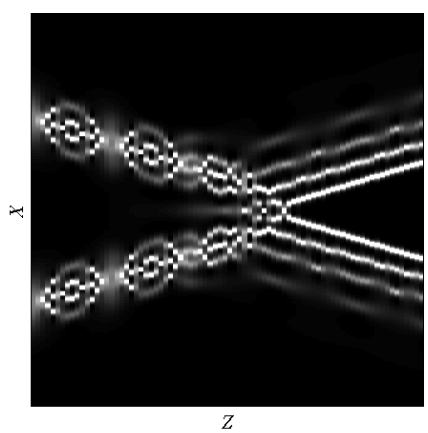

Fig. 4. Top view of the symmetric collision of two solitons of fifth order for the same conditions than Fig. 3.

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4. Symmetric collision of different order bright spatial solitons

In this section we present the results obtained for the collision between solitons of different order. This means that the initial condition was of the form:

$$q(X, Z=0) = N_1 \sec h(X - X_C)\exp(-iVX) + N_2 \sec h(X + X_C)\exp(iVX), \quad (5)$$

where N_1 and N_2 are positive integers and $N_2 > N_1$.

When one of the colliding solitons is a fundamental of order one, after the collision the high order soliton does not fully split. However, it presents a beam with a modulation on its top with the same number of peaks that its order, see Fig. 5. Larger propagation distances and transversal velocities do not affect considerably this result. We can conclude that the collision with a fundamental spatial soliton does not split a high order soliton.

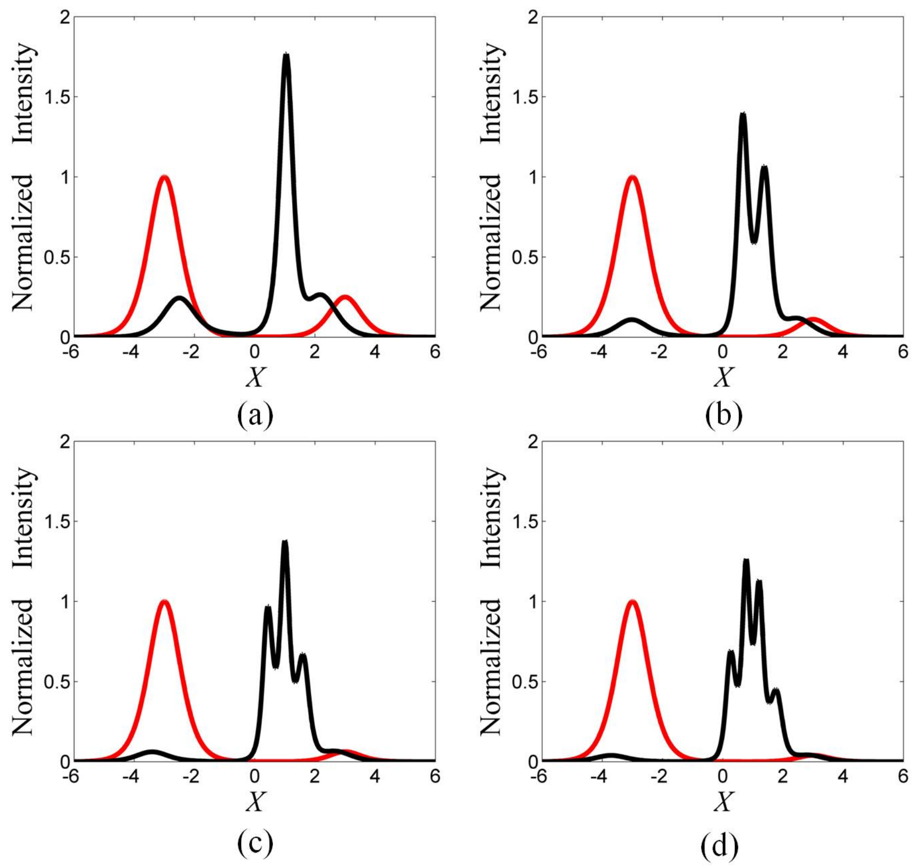

Fig. 5. Intensity profiles at $Z = 0$ (red) and $Z = 5\pi/2$ (black) for the symmetric collision between a fundamental soliton and a high order soliton of: (a) 2nd, (b) 3th, (c) 4th and (d) 5th.

When both of the interacting solitons are of order larger than two, then after the collision it is possible to obtain the splitting of the solitons in a number of beams that correspond to the order of the lowest order colliding soliton, see Figs. 6 to 8. From this and the previous results we can conclude that the splitting of a high order spatial soliton is obtained when the collision is with another soliton of the same or larger order.

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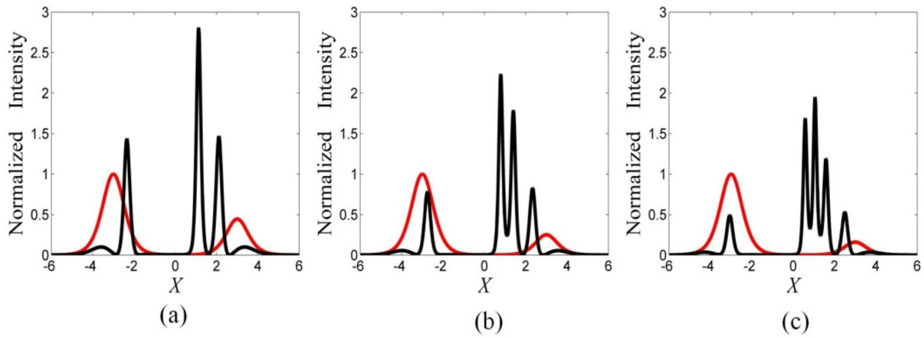

Fig. 6. Intensity profiles at $Z = 0$ (red) and $Z = 5\pi/2$ (black) for the symmetric collision between a second order spatial soliton and a high order soliton of: (a) 3th, (b) 4th and (c) 5th.

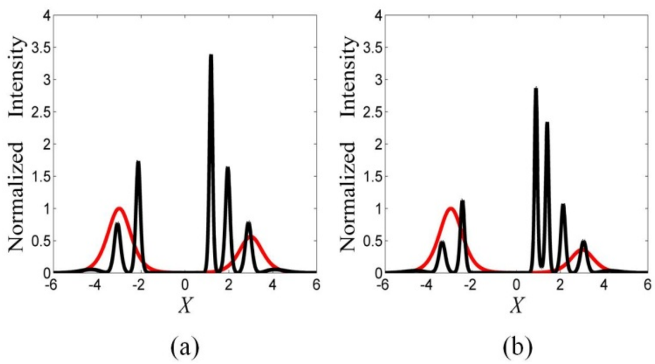

Fig. 7. Intensity profiles at $Z = 0$ (red) and $Z = 5\pi/2$ (black) for the symmetric collision between a third order spatial soliton and a high order soliton of: (a) 4th and (b) 5th.

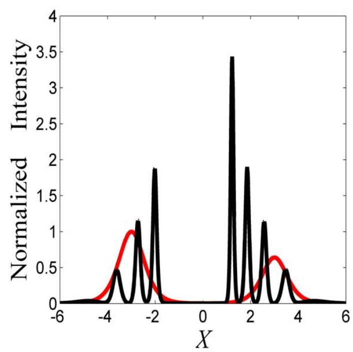

Fig. 8. Intensity profiles at $Z = 0$ (red) and $Z = 5\pi/2$ (black) for the symmetric collision between a fifth order and a forth order spatial soliton.

5. Collision of a high order soliton with its virtual image at a linear interface

A linear interface was simulated considering the following function:

$$f(x) = \frac{1}{2}(1 - \tan\mathrm{h}\,\kappa x), \tag{6}$$

where κ is the steepness of the interface. For x<0 the medium was considered nonlinear Kerr type (as in the previous sections) with a linear refractive index n_0. For x>0 the medium was

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considered linear with a linear refractive index n_1. The first studies on solitons incident to interfaces were made in 1989 [27]. Since then many contributions to understand the phenomena have been done.

In our case $n_0 > n_1$ such that a fundamental soliton can be totally reflected at some critical angle ($V = 0.3$, see Fig. 9(a)). Setting the interface at $X = 2$ and the high order soliton at $X = 0$ with a transversal velocity of $V = 0.3$, under these conditions the splitting of the high order soliton was obtained after reflection, see Fig. 9. The number of beams obtained after reflection coincide with the soliton order. The angle with respect to the interface of the reflected beams is not the same: the more intense beam made the same angle that the incident soliton, the other beams made a larger one. The intensity of the reflected beams decays as their distance from the interface. The high intensity reflected beam behaved as a fundamental soliton, the rest presented an oscillatory behavior. Smaller angles of the soliton to the interface produced a more complicated dynamic in the reflected beams but the splitting was always obtained.

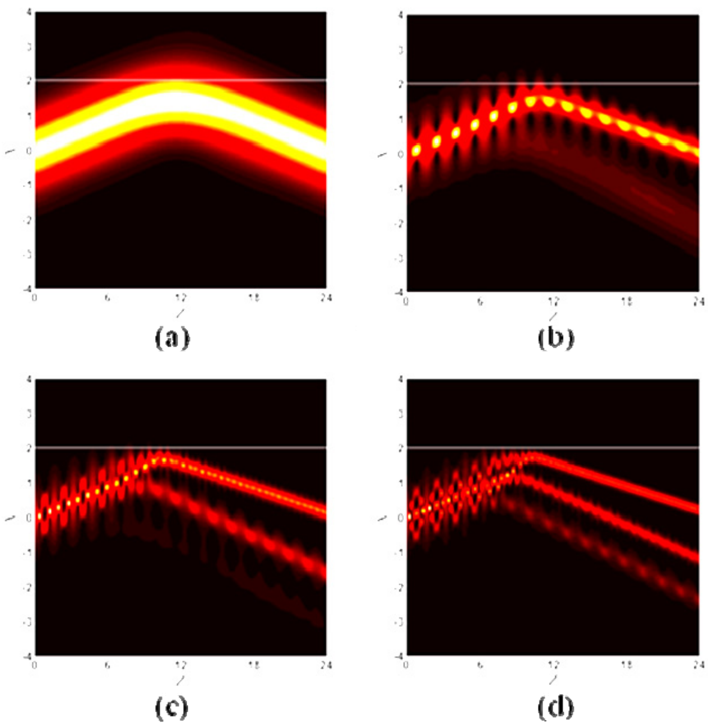

Fig. 9. High order solitons incident to a linear interface. Interface set at $X = 2$. Soliton transversal velocity $V = 0.3$. Propagation distance of 24. Soliton order of (a) fundamental, (b) 2nd, (c) 3th and (d) 4 $^{\text{th}}$.

6. Conclusion

We have demonstrated the splitting of high order bright spatial solitons after collision with another high order soliton or a virtual one created by the reflection in a linear interface. For collisions between spatial solitons of the same order, the results demonstrate that after the collision the high order soliton split in a number of beams that correspond to the order of the soliton. Collision between solitons of different order does not necessary split the soliton of the higher order. In order to obtain the splitting it is not necessary to make a collision with other soliton, the splitting can be stimulated by the reflection at a linear. The results presented in this work can be applied for the case of temporal solitons.

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