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Research Article

Partial Polarization in Interfered Plasmon Fields

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We describe the polarization features for plasmon fields generated by the interference between two elemental surface plasmon modes, obtaining a set of Stokes parameters which allows establishing a parallelism with the traditional polarization model. With the analysis presented, we find the corresponding coherence matrix for plasmon fields incorporating to the plasmon optics the study of partial polarization effects.

1. Introduction

Elemental surface plasmon modes (Sp) are nonhomogeneous vector waves generated by the coherent oscillations of the plasma electrons [1, 2]. A generic feature that presents vector waves is the polarization which can be understood as the study of the trajectory that describes the electric field as a function of the amplitude and relative phases [3]. As a consequence of the vector structure of the Sp, polarization features can be introduced in a similar way to homogeneous waves propagating in free space. However, deep physical differences must be remarked. One of them is that the path to the electric field shares the same plane with the Poynting vector. In fact, each component of the electric field for the Sp is related with each other, according to Maxwell equations; this means that no term can be individually manipulated without modifying the other components. As a consequence, the polarization state for Sp is fixed and the polarization path cannot be modified without changing the structure of the Sp. Previous statements are contained in the expression for the electric field of a Sp, given by

$$\vec{E}_1 = a\left(\hat{i} + \frac{\alpha_1}{i\beta}\hat{k}\right)e^{i(\beta z - wt)}e^{-\alpha_1 x}, \quad x \geq 0;$$

$$\vec{E}_2 = a\left(\frac{\varepsilon_1}{\varepsilon_2}\hat{i} - \frac{\varepsilon_1\alpha_2}{\varepsilon_2 i\beta}\hat{k}\right)e^{i(\beta z - wt)}e^{\alpha_2 x}, \quad x < 0, \tag{1}$$

where the subindex refers to the parameters of the electric field in each media. In this representation, $\varepsilon_{1,2}$ are the dielectric constant on each media, a is a parameter that describes the amplitude of the Sp, $\alpha_{1,2}$ are the decaying factor, and β is the dispersion relation function.

In the present work, we develop the study of plasmon fields, whose structure can be tunable, allowing incorporating the traditional model of polarization. We propose to use a superposition of surface plasmon cosine modes (Spc) generated by the interference between two Sp, where the interference parameters can be controlled [4], incorporating in this way, the polarization features desired. The convergence of the set of Spc generates plasmon fields with partial coherence and consequently with partial polarized features. From this propose the Stokes parameters and consequently the coherence matrix for plasmon fields can be obtained by letting the interference parameters have the behavior of a random variable, where the probability density function allows the calculus of the mean values.

The study presented represents a step forward in the study of the physics of the plasmon fields offering a great variety of contemporary applications in topics such as tunable spectroscopy, the development of plasmon tweezers, and synthesis of metamaterials.

2. Theory

The general structure of a Sp, propagating on the plane y-z, can be obtained from (1), applying a rotation along x-axis, using the expression for the electric field in medium 1, it acquires the form:

$$\vec{E} = a\left(\hat{i} + \hat{j}\frac{i\alpha}{\beta}\sin\theta + \hat{k}\frac{i\alpha}{\beta}\cos\theta\right)e^{-\alpha x}e^{i\beta(z\cos\theta + y\sin\theta)}, \quad (2)$$

where we have avoided the subindex to simplify the notation.

With the purpose to identify the interference parameters, which allows to control the polarization features, we describe the sum between two Sp whose expression is given by

$$\begin{aligned}\vec{E} &= a\left(\hat{i} + \hat{j}\frac{i\alpha}{\beta}\sin\theta + \hat{k}\frac{i\alpha}{\beta}\cos\theta\right)e^{-\alpha x}e^{i\beta(z\cos\theta + y\sin\theta)} \\ &\quad + a\left(\hat{i} - \hat{j}\frac{i\alpha}{\beta}\sin\theta + \hat{k}\frac{i\alpha}{\beta}\cos\theta\right)e^{-\alpha x}e^{i\beta(z\cos\theta - y\sin\theta)}.\end{aligned} \quad (3)$$

Rewriting the resulting vector as a column vector, we obtain

$$\begin{aligned}\vec{E} &= 2ae^{-\alpha x}e^{i(\mathrm{Re}\,\beta)z\cos\theta} \\ &\quad \times e^{-(\mathrm{Im}\,\beta)z\cos\theta}\begin{pmatrix}\cos(\mathrm{Re}\,\beta y\sin\theta) \\ W\sin(\mathrm{Re}\,\beta y\sin\theta)\sin\theta e^{i\delta_y} \\ W\cos(\mathrm{Re}\,\beta y\sin\theta)\cos\theta e^{i\delta_z}\end{pmatrix},\end{aligned} \quad (4)$$

whose structure corresponds to a plasmon beam cosine kind (Spc); here we have the parameters

$$W = |\beta|, \quad \delta_y = \frac{\mathrm{Im}\beta}{\mathrm{Re}\,\beta}, \qquad \delta_z = \frac{\mathrm{Im}\beta}{\mathrm{Re}\,\beta} + \frac{\pi}{2}. \quad (5)$$

To describe the polarization effects, from (4), we define the Jones plasmon vector as

$$\vec{J} = \begin{pmatrix}J_x \\ J_y \\ J_z\end{pmatrix} = \begin{pmatrix}\cos\Omega \\ W\sin\Omega\sin\theta e^{i\delta_y} \\ W\cos\Omega\cos\theta e^{i\delta_z}\end{pmatrix}, \quad (6)$$

where $\Omega = \mathrm{Re}\,\beta y\sin\theta$. Analyzing the projection of the Jones plasmon vector on each plane and following the classical treatment of polarization [3], we identify three paths to the electric field on (x-y), (x-z), and (y-z) planes as follows.

(1) On the (x-y) plane, the trajectory for the electric field is

$$\begin{aligned}&\left(\frac{E_x}{\cos\Omega}\right)^2 + \left(\frac{E_y}{W\sin\theta\sin\Omega}\right)^2 \\ &\quad - \left(\frac{2E_xE_y}{W\cos\Omega\sin\Omega\sin\theta}\right)\cos\delta_y = \left(\sin\delta_y\right)^2,\end{aligned} \quad (7)$$

and the corresponding Stokes parameters are

$$\begin{aligned}S_{0xy} &= (\cos\Omega)^2 + W^2(\sin\theta)^2(\sin\Omega)^2, \\ S_{1xy} &= (\cos\Omega)^2 - W^2(\sin\theta)^2(\sin\Omega)^2, \\ S_{2xy} &= 2W\cos\Omega\sin\theta\sin\Omega\cos\delta_y, \\ S_{3xy} &= 2W\cos\Omega\sin\theta\sin\Omega\sin\delta_y.\end{aligned} \quad (8)$$

Here we must note that Stokes parameters are dependent on the y coordinate. The simplest case corresponding to the zero order interference fringes is obtained when $y = 0$; the values for the Stokes parameters are

$$\left(S_{0xy}, S_{1xy}, S_{2xy}, S_{3xy}\right)_{y=0} = (1, 1, 0, 0). \quad (9)$$

This means that the zero order interference fringe has linear polarization, being the electric field perpendicular to the y-z plane.

(2) On the (x-z) plane, the trajectory is

$$\begin{aligned}&\left(\frac{E_x}{\cos\Omega}\right)^2 + \left(\frac{E_z}{W\cos\theta\cos\Omega}\right)^2 \\ &\quad - \left(\frac{2E_xE_z}{W\cos\theta(\cos\Omega)^2}\right)\cos\delta_z = (\sin\delta_z)^2.\end{aligned} \quad (10)$$

The Stokes parameters are given by

$$\begin{aligned}S_{0xz} &= (\cos\Omega)^2 + W^2(\cos\theta)^2(\cos\Omega)^2, \\ S_{1xz} &= (\cos\Omega)^2 - W^2\left(\cos\theta\right)^2(\cos\Omega)^2, \\ S_{2xz} &= 2W(\cos\Omega)^2\cos\theta\cos\delta_z, \\ S_{3xz} &= 2W(\cos\Omega)^2\cos\theta\sin\delta_z.\end{aligned} \quad (11)$$

When $y = 0$, the Stokes parameters take the form

$$\begin{aligned}&(S_{0xz}, S_{1xz}, S_{2xz}, S_{3xz})_{y=0} \\ &\quad = \left(1 + W^2\cos\theta^2, 1 - W^2\cos\theta^2,\right. \\ &\qquad \left. 2W\cos\theta\cos\delta_z, 2W\cos\theta\sin\delta_z\right),\end{aligned} \quad (12)$$

which means that the zero order interference fringe on (x-z) plane has elliptical polarization.

(3) On the (y-z) plane, the trajectory is

$$\begin{aligned}&\left(\frac{E_y}{W\sin\Omega\sin\theta}\right)^2 + \left(\frac{E_z}{W\cos\Omega\cos\theta}\right)^2 \\ &\quad - \left(\frac{2E_yE_z}{W^2\sin\Omega\sin\theta\cos\Omega\cos\theta}\right)\cos\delta_z = (\sin\delta_z)^2.\end{aligned} \quad (13)$$

The Stokes parameters are given by

$$
\begin{aligned}
S_{0yz} &= W^2(\sin\theta)^2(\sin\Omega)^2 + W^2(\cos\theta)^2(\cos\Omega)^2,\\
S_{1yz} &= W^2(\sin\theta)^2(\sin\Omega)^2 - W^2(\cos\theta)^2(\cos\Omega)^2,\\
S_{2yz} &= 2W^2\sin\theta\sin\Omega\cos\theta\cos\Omega\cos\delta_z,\\
S_{3yz} &= 2W^2\sin\theta\sin\Omega\cos\theta\cos\Omega\sin\delta_z.
\end{aligned}
\tag{14}
$$

On $y = 0$, the values for the Stokes parameters are

$$
\left(S_{0yz}, S_{1yz}, S_{2yz}, S_{3yz}\right)_{y=0} = \left(W^2\cos\theta^2, -W^2\cos\theta^2, 0, 0\right), \tag{15}
$$

which means that the electric field has linear polarization along z-coordinate.

It is easy to show that the Stokes parameters on each plane satisfy

$$
S_0^2 = S_1^2 + S_2^2 + S_3^2, \tag{16}
$$

which is satisfied for completely coherent and polarized plasmon fields. Finally we remark that each set of Stokes parameter depends on (y, θ) parameters. The parameter θ will be used in the following section to generate a set of Spc modes mutually incoherent whose convergence generates partially polarized plasmon fields [4].

As a partial conclusion, we get that the elemental surface plasmon has a fixed polarization state; however, the interference between two of them presents similar features to classical optical polarizable fields. The polarization state is described by projecting the electric field on three mutually perpendicular planes.

3. Partially Polarized Surface Plasmon Modes

The set of Stokes parameters for interfered beams are dependent on the (y, θ) parameters; consequently, each interference fringe has different polarization states and they have associated three Poincare's spheres corresponding to the projected plasmon electric field over each plane. From this representation it is possible to incorporate partially polarized effects implicit in the coherence matrix associated to each plane. The structure of the coherence matrix in terms of the Stokes parameters is [5]

$$
J = \begin{bmatrix} \left\langle \dfrac{S_0 + S_1}{2} \right\rangle & \left\langle \dfrac{S_2 + iS_3}{2} \right\rangle \\ \left\langle \dfrac{S_2 - iS_3}{2} \right\rangle & \left\langle \dfrac{S_0 - S_1}{2} \right\rangle \end{bmatrix}, \tag{17}
$$

and the angle brackets represent the mean value; it is obtained using the relation

$$
\langle S_i \rangle = \int S_i(y,\theta)\,\rho(\theta)\,d\theta = M_i(y); \quad i = 0,1,2,3, \tag{18}
$$

where $\rho(\theta)$ is the probability density function. For the experimental implementation, we propose to use a gold thin film containing two subwavelength holes [6, 7] deposited on an elastic material, applying a random force parallel to the y-axis that connects the holes, we can control the relative separation between them to generate an ensemble of Spc. The setup to generate the interference is sketched in Figure 1, where the oil drop acts as a lens, generating the sum between two Sp. More details can be founded in [8].

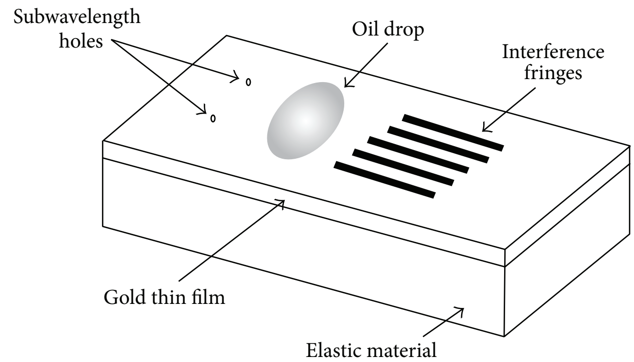

Figure 1: Experimental setup to generate the incoherent convergence of Spc. The width of the gold thin film is 40 nm placed on an elastic material. The relative position between holes is in the interval 5–15 nm and it is controlled applying a random force. The refractive index for the oil drop is $n = 2.1$.

As an example we consider the case when the probability density function $\rho(\theta)$ is uniform in the interval $-\theta_0 \le \theta \le \theta_0$. Expanding the Stokes parameters given by (8)–(14) in terms of Bessel function [9], the mean values acquire the following forms.

On the $(x\text{-}y)$ plane the Stokes parameters are

$$
\begin{aligned}
\langle S_{0xy} \rangle &= \frac{1}{2} + \left(\frac{1}{2} - \frac{W^2}{4}\right) J_0(2\,\mathrm{Re}\,\beta y) + \frac{W^2}{4} J_2(2\,\mathrm{Re}\,\beta y),\\
\langle S_{1xy} \rangle &= \frac{1}{2} + \left(\frac{1}{2} + \frac{W^2}{4}\right) J_0(2\,\mathrm{Re}\,\beta y) - \frac{W^2}{4} J_2(2\,\mathrm{Re}\,\beta y),\\
\langle S_{2xy} \rangle &= 2W\cos\delta_y J_1(2\,\mathrm{Re}\,\beta y),\\
\langle S_{3xy} \rangle &= 2W\sin\delta_y J_1(2\,\mathrm{Re}\,\beta y).
\end{aligned}
\tag{19}
$$

In general, the mean polarization on $(x\text{-}y)$ plane corresponds to elliptical polarization, containing the case of linear polarization which occurs when $y = 0$.

On the $(x\text{-}z)$ plane the Stokes parameters are

$$
\langle S_{0xz} \rangle = \left(\frac{1}{2} + \frac{W^2}{4}\right) J_0(2\,\mathrm{Re}\,\beta y) + \frac{W^2}{4} J_2(2\,\mathrm{Re}\,\beta y) + \frac{1}{2} + \frac{W^2}{4},
$$

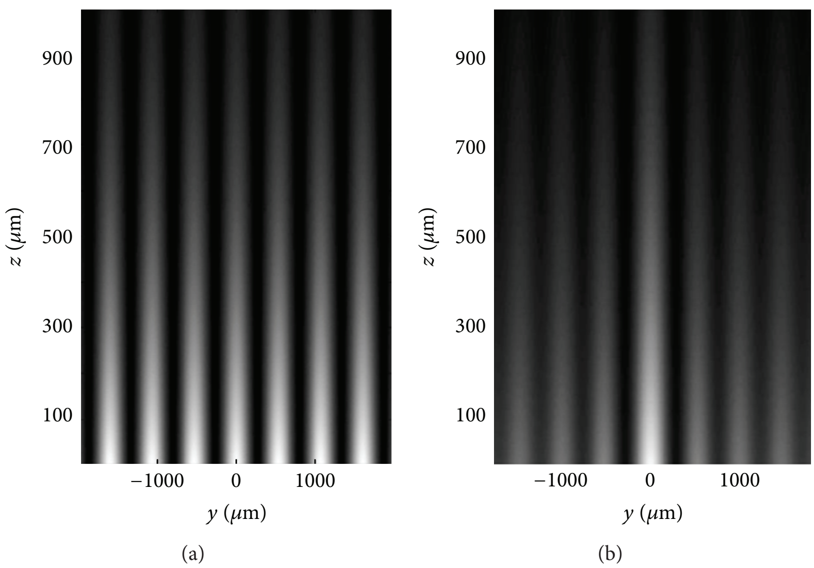

FIGURE 2: (a) Irradiance distribution for the cosine mode completely coherent. (b) Partially polarized plasmon mode with uniform probability density function.

$$\begin{aligned}\langle S_{1xz}\rangle &= \left(\frac{1}{2}+\frac{W^2}{4}\right)J_0\left(2\operatorname{Re}\beta y\right)\\ &\quad-\frac{W^2}{4}J_2\left(2\operatorname{Re}\beta y\right)+\frac{1}{2}-\frac{W^2}{4},\\ \langle S_{2xz}\rangle &= W\cos\delta_z J_0\left(2\operatorname{Re}\beta y\right),\\ \langle S_{3xz}\rangle &= W\sin\delta_z J_0\left(2\operatorname{Re}\beta y\right).\end{aligned} \tag{20}$$

In general, the mean polarization on (x-z) plane corresponds to elliptical polarization.

On the (y-z) plane the Stokes parameters are

$$\begin{aligned}\left\langle S_{0yz}\right\rangle &= W^2\left(1+J_0\left(2\operatorname{Re}\beta y\right)\right)+J_2\left(2\operatorname{Re}\beta y\right),\\ \left\langle S_{1yz}\right\rangle &= 0,\\ \left\langle S_{2yz}\right\rangle &= 0,\\ \left\langle S_{3yz}\right\rangle &= 0.\end{aligned} \tag{21}$$

On (y-z) plane the plasmon field is completely nonpolarized.

In Figure 2(a) we show the computer simulation for the irradiance distribution on (y-z) plane associated to the surface plasmon cosine mode completely coherent associated with (4). In Figure 2(b), we show the computer simulation when the relative separation between apertures, sketched in Figure 1, follows a uniform probability density function, where the modulation curve is easily identified. The calculus was obtained by taking the square modulus of (4) and then obtaining the mean value. We select the (y-z) plane because it is matched with the metal surface which allows us to establish the reference system.

4. Final Remarks and Conclusions

The elemental surface plasmon mode has a polarization state which cannot be modified; for this reason, in the present study, we use plasmon interfered modes, where the parameters responsible of the interference allow introducing partial polarization effects. In addition, with this kind of plasmon fields, it is possible to induce arrays of particles whose distributions depend on the interference fringes and the plasmon polarization effects are capable of inducing the particles tunable dipole moments. We remark that the dipole moments of the particle depend on the interference fringe, generating variations in the refractive index depending on position, which offers applications to the synthesis of tunable materials, in particular, the generation of metamaterials; another application is the generation of plasmon percolation effects by propagating the dipolar wave trough the particles arrays. In other context, the partial polarized features can be implemented for particle trapping [10], also as the generation of plasmon tweezers and tunable spectroscopy features.

The study presented can be extended in a general way by implementing different probability density functions, and the integral for the calculus of the mean values given by (18) can be reinterpreted as a first kind Fredholm integral equation [11], whose kernel is the Stokes parameters. This can be done by proposing a specific function for $M_i(y)$, where now the unknown function is the probability density function $\rho(\theta)$.

As conclusions, we performed the study of the plasmon electric field establishing an analogy with the traditional model of polarization. The study was implemented by means of the interference between elemental surface plasmon modes, whose parameters are tunable allowing us to induce partially polarized features. The electric field was projected on three mutual perpendicular planes and paves the way to get a deep understanding about the conditions under in which radiative process in plasmon fields may occur also as the generation of plasmon polarization singularities [12], which must occur in the plasmon focusing regions [13].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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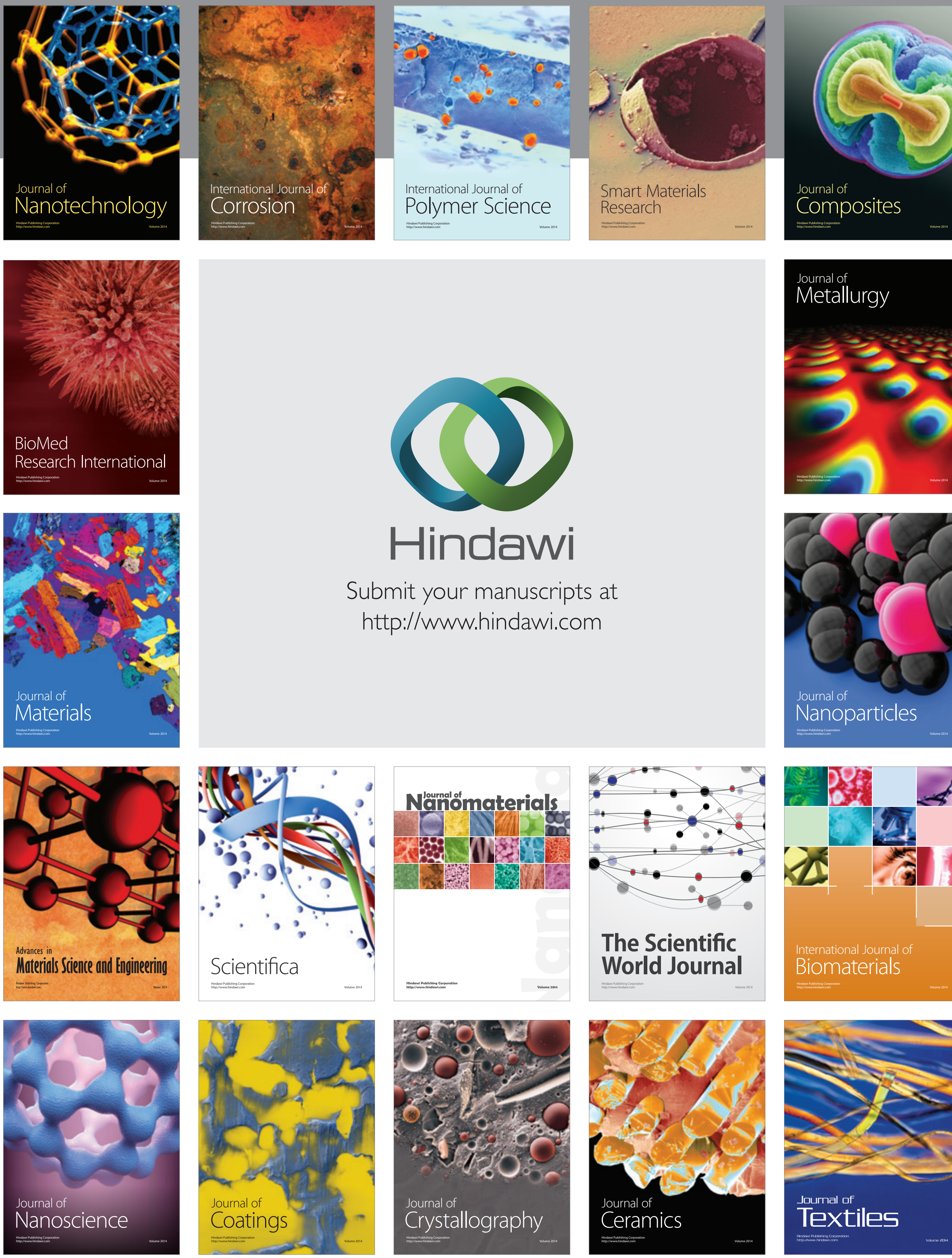
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