Experimental determining the coherent-mode structure of vector electromagnetic field through its decomposition in reference basis

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Abstract: A technique for experimental determining the coherent-mode structure of electromagnetic field is proposed. This technique is based on the coherence measurements of the field in some reference basis and represents a nontrivial vector generalization of the dual-mode field correlation method recently reported by F. Ferreira and M. Belsley [Opt. Lett. **38**(21), 4350 (2013)]. The justifiability and efficiency of the proposed technique is illustrated by an example of determining the coherent-mode structure of some specially generated and experimentally characterized secondary electromagnetic source.

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References and links

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1. Introduction

The coherent-mode representation of an optical field broached the first time by Gamo [1] and later on developed by Wolf [2–4] is an essential tool in describing the processes and systems in optics [5]. Not so long ago the theory of coherent-mode representation, originally developed for scalar optical fields, has been generalized to the case of vector electromagnetic fields [6–8]. This representation is defined through the solution of the Fredholm integral equation with a kernel taken as the cross-spectral density matrix of the field. However, in practice the cross-spectral density matrix of the field as a rule is unknown *a priori*. In theory the cross-spectral density matrix of the field can be measured with four special Young's interference experiment [9,10], but in practice such a measurement is unfeasible in view of enormous volume of data to be processed.

Recently a new promising approach to the problem of experimental determining the coherent-mode structure of a scalar optical field has been proposed by F. Ferreira and M. Belsley [11]. This approach is based on the decomposition of a scalar field in some subsidiary orthogonal basis, which allows considerable simplification of the coherence measurements process. Here we propose the generalization of this approach to the case of a vector electromagnetic field. The justifiability and efficiency of the proposed technique is illustrated by an example of determining the coherent-mode structure of some specially generated and experimentally characterized secondary electromagnetic source.

2. Coherent-mode structure of electromagnetic field

We start recalling the basic concepts of the theory of partially coherent electromagnetic fields in the space-frequency domain [7]. According to this theory the second-order statistical properties of a stochastic stationary electromagnetic field occupying some finite domain D in some plane normal to the direction of propagation and at some frequency ν may be completely characterized by the so-called cross-spectral density matrix (for brevity we omit the explicit dependence of the considered quantities on ν)

$$\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)=\begin{bmatrix} W_{xx}(\mathbf{x}_1,\mathbf{x}_2) & W_{xy}(\mathbf{x}_1,\mathbf{x}_2) \\ W_{yx}(\mathbf{x}_1,\mathbf{x}_2) & W_{yy}(\mathbf{x}_1,\mathbf{x}_2) \end{bmatrix}, \tag{1}$$

where

$$W_{ij}(\mathbf{x}_1,\mathbf{x}_2)=\left\langle E_i^*(\mathbf{x}_1)E_j(\mathbf{x}_2)\right\rangle, \quad (i,j=x,y), \tag{2}$$

with E_i and E_j being the orthogonal components of the electric field vector $\mathbf{E}$ at two points $\mathbf{x}_1$ and $\mathbf{x}_2$, asterisk denoting the complex conjugate, and the angle brackets denoting the average over the statistical ensemble. Furthermore, the correlation properties of a partially coherent and partially polarized electromagnetic field may be characterized quantitatively using the degree of coherence and degree of polarization defined by the formulas, respectively,

$$\eta(\mathbf{x}_1,\mathbf{x}_2)=\left(\frac{\mathrm{Tr}[\mathbf{W}^\dagger(\mathbf{x}_1,\mathbf{x}_2)\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)]}{\mathrm{Tr}\mathbf{W}(\mathbf{x}_1,\mathbf{x}_1)\mathrm{Tr}\mathbf{W}(\mathbf{x}_2,\mathbf{x}_2)}\right)^{1/2}, \tag{3}$$

$$P(\mathbf{x})=\left(1-\frac{4\mathrm{Det}\mathbf{W}(\mathbf{x},\mathbf{x})}{\left[\mathrm{Tr}\mathbf{W}(\mathbf{x},\mathbf{x})\right]^2}\right)^{1/2}, \tag{4}$$

where Tr stands for the trace, Det denotes the determinant of matrix, and the dagger denotes the Hermitian conjunction. As has been shown in [7], under very general conditions the cross-spectral density matrix $\mathbf{W}$ may be represented in the form of series, i.e.,

$$\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)=\sum_n \lambda_n \mathbf{W}_n(\mathbf{x}_1,\mathbf{x}_2), \quad (n=0,1,2,...), \tag{5}$$

where $\mathbf{W}_n(\mathbf{x}_1,\mathbf{x}_2)$ is the 2×2 matrix with elements

$$W_{ij;n}(\mathbf{x}_1,\mathbf{x}_2)=\varphi_{i;n}^*(\mathbf{x}_1)\varphi_{j;n}(\mathbf{x}_2). \tag{6}$$

In Eq. (5) λ_n and $\varphi_n^{(i)}(\mathbf{x})$ are the eigenvalues and the eigenfunctions of two coupled integral equations

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$$\sum_j \int_D W_{ij;n}(\mathbf{x}_1,\mathbf{x}_2)\varphi_{j;n}(\mathbf{x}_1)\mathrm{d}\mathbf{x}_1 = \lambda_n \varphi_{i;n}(\mathbf{x}_2). \tag{7}$$

The eigenvalues are real and nonnegative, and the eigenfunctions satisfy the orthonormality condition

$$\int_D \varphi^*_{i;n}(\mathbf{x})\varphi_{i;m}(\mathbf{x})\mathrm{d}\mathbf{x} = \delta_{nm}, \tag{8}$$

where δ_{nm} is the Kronecker symbol. Each matrix $\mathbf{W}_n(\mathbf{x}_1,\mathbf{x}_2)$ in Eq. (5) can be associated with an elementary mode of the field which is completely coherent ($\eta_n = 1$) and completely polarized ($P_n = 1$).Therefore the set of λ_n and $\varphi_{i;n}(\mathbf{x})$ is referred to as the coherent-mode structure of the field.

3. Decomposition of the coherent-mode structure in reference basis

Now, adopting the main idea of [11] originally formulated for a scalar field, we will show that the coherent-mode structure of a vector electromagnetic field may be defined in a more practical way. To do this, we assume that the realizations of each orthogonal component E_i of the electric field vector can be expanded in some orthogonal basis $\{\psi_k(\mathbf{x})\}$, which we will refer to the reference basis, as follows:

$$E_i(\mathbf{x}) = \sum_k a_{i;k}\psi_k(\mathbf{x}), \tag{9}$$

$$\int_D \psi^*_k(\mathbf{x})\psi_l(\mathbf{x})\mathrm{d}\mathbf{x} = \delta_{kl}, \tag{10}$$

$$a_{i;k} = \int_D E_i(\mathbf{x})\psi^*_k(\mathbf{x})\mathrm{d}\mathbf{x}. \tag{11}$$

Substituting for E_i from Eq. (9) into Eq. (2), we obtain

$$W_{ij}(\mathbf{x}_1,\mathbf{x}_2) = \sum_k\sum_l c_{ij;kl}\psi^*_k(\mathbf{x}_1)\psi_l(\mathbf{x}_2), \tag{12}$$

where

$$c_{ij;kl} = \left\langle a^*_{i;k}a_{j;l}\right\rangle. \tag{13}$$

Substituting from Eq. (12) into Eq. (7), we find

$$\sum_j\sum_k\sum_l c_{ij;kl}b_{j;n;k}\psi_l(\mathbf{x}) = \lambda_n\varphi_{i;n}(\mathbf{x}), \tag{14}$$

where

$$b_{j;n;k} = \int_D \varphi_{j;n}(\mathbf{x})\psi^*_k(\mathbf{x})\mathrm{d}\mathbf{x}. \tag{15}$$

Finally, multiplying both sides of Eq. (14) by $\psi^*_s(\mathbf{x})$ and integrating the result over $\mathbf{x}$ with due regard for the orthogonality relation (10), we obtain the system of algebraic equations

$$\sum_j\sum_k c_{ij;kl}b_{j;n;k} = \lambda_n b_{i;n;l}. \tag{16}$$

This system can be written in matrix form as follows:

$$\begin{bmatrix}\mathbf{C}_{xx} & \mathbf{C}_{xy}\\ \mathbf{C}^\dagger_{xy} & \mathbf{C}_{yy}\end{bmatrix}\begin{bmatrix}\mathbf{B}_{x;n}\\ \mathbf{B}_{y;n}\end{bmatrix} = \lambda_n\begin{bmatrix}\mathbf{B}_{x;n}\\ \mathbf{B}_{y;n}\end{bmatrix}, \tag{17}$$

where $\mathbf{C}_{ij}$ is the square matrix with elements given by Eq. (13) and $\mathbf{B}_{i;n}$ is the column matrix with elements given by Eq. (15). The eigenvalues λ_n can be found by solving the characteristic equation

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$$\mathrm{Det}\left(\begin{bmatrix}\mathbf{C}_{xx} & \mathbf{C}_{xy}\\ \mathbf{C}_{xy}^{\dagger} & \mathbf{C}_{yy}\end{bmatrix}-\lambda_n\mathbf{I}\right)=0. \tag{18}$$

By virtue of definition (13) the matrix $\mathbf{C}$ composed of sub-matrices $\mathbf{C}_{ij}$ is Hermitian ($\mathbf{C}^{\dagger} = \mathbf{C}$), assuring that all eigenvalues will be real as stated in the previous section. Once the eigenvalues λ_n have been found, one can calculate the coefficients $b_{i;n;k}$ solving the corresponding system given by Eq. (17). Then, the unknown modal functions $\varphi_{i;n}(\mathbf{x})$ can be determined in form of the expansions

$$\varphi_{i;n}(\mathbf{x})=\sum_k b_{i;n;k}\psi_k(\mathbf{x}). \tag{19}$$

It must be noted that in practice one needs truncating the complete reference basis by finite number K of functions $\psi_k(\mathbf{x})$, which depends on the field complexity and the degree of mismatch between the reference basis and actual modal basis of the field [11]. In fact, the number K is limited by the admissible complexity of field correlation measurements (see next section) and cannot exceed a few tens. On the other hand, K determines the number of sought-for coherent modes $\varphi_n(\mathbf{x})$. As well known [12], the effective number of coherent modes depends on the degree of coherence of the field and can take a very great value for rather incoherent field. Thus, the proposed technique may be effectively used only for fairly coherent fields. At first sight this circumstance restricts seriously our technique in its possible applications. However, we remark that a field with a low enough degree of coherence frequently can be considered approximately as completely incoherent, when the concept of the coherent-mode structure and, hence, the proposed technique lose in general their practical sense.

4. Measurement of matrix $\mathbf{C}$

To solve Eq. (17), the coefficients $c_{ij;kl}$ must be known. Below we show that these coefficients can be measured by means of the modified Mach-Zehnder interferometer sketched schematically in Fig. 1.

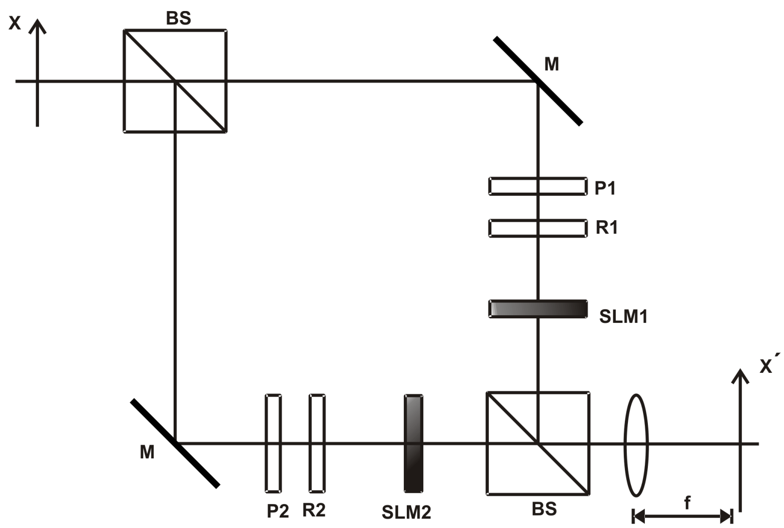

Fig. 1. Optical system for measuring coefficients $c_{ij;kl}$: BS, beam splitter; M, mirror; P, polarizer; R, polarization rotator; SLM, spatial light modulator; L, lens.

Let us consider that the electromagnetic field at the input of interferometer is characterized by the electric field vector $\mathbf{E}$, and let polarizers P1 and P2 be chosen to transmit only one orthogonal component E_x or E_y. The polarization rotators R1 and R2 serve to align the polarization planes of the selected orthogonal components allowing their posterior interference. The spatial light modulators SLM1 and SLM2 modify independently the amplitudes of selected components. The lens L projects the Fourier transform of the incident field onto its back focal plane.

Let the amplitude transmittance of each spatial light modulator be described by

$$t_k(\mathbf{x}) = t_0 + 2\left|\psi_k(\mathbf{x})\right|\cos\left[\operatorname{Arg}\left(\psi_k(\mathbf{x})\right) + 2\pi x p_0 + \beta_k\right], \tag{20}$$

where t_0 is a constant chosen to provide the non-negativity of $t_k(\mathbf{x})$, and p_0 and β_k are the constants whose meaning will be defined below. The amplitude of the field in the back focal plane of lens L is given by

$$U_{ij;kl}(\mathbf{x}') = \int_D \left[E_i(\mathbf{x})t_k(\mathbf{x}) + E_j(\mathbf{x})t_l(\mathbf{x})\right]\exp\left(i\frac{2\pi}{\lambda f}\mathbf{x}'\cdot\mathbf{x}\right)\mathrm{d}\mathbf{x}, \tag{21}$$

where λ is the wavelength of illumination and f is the lens focal distance. Then, substituting from Eq. (20) into Eq. (21) with due regard of notation (11), one finds that the field amplitude at the specific point $\mathbf{x}_0' = (\lambda f p_0, 0)$ is as follows:

$$U_{ij;kl} = a_{i;k}\exp(-i\beta_k) + a_{j;l}\exp(-i\beta_l). \tag{22}$$

Thus, the average intensity of the field at this point with due regard of notation (13) appears to be

$$I_{ij;kl}(\beta_{kl}) = \left\langle \left|U_{ij;kl}\right|^2 \right\rangle = c_{ii;kk} + c_{jj;ll} + c_{ij;kl}\exp(i\beta_{kl}) + c_{ij;kl}^*\exp(-i\beta_{kl}), \tag{23}$$

where $\beta_{kl} = \beta_k - \beta_l$. It can be readily shown that, measuring the intensity given by Eq. (23) for two particular values $\beta_{kl} = 0$ and $\beta_{kl} = -\pi/2$, one can find the real and imaginary parts of $c_{ij;kl}$ as follows:

$$\operatorname{Re}(c_{ij;kl}) = \frac{1}{2}I_{ij;kl}(0) - \frac{1}{8}I_{ii;kk}(0) - \frac{1}{8}I_{jj;ll}(0), \tag{24}$$

$$\operatorname{Im}(c_{ij;kl}) = \frac{1}{2}I_{ij;kl}(-\pi/2) - \frac{1}{8}I_{ii;kk}(0) - \frac{1}{8}I_{jj;ll}(0). \tag{25}$$

Taking into account the Hermitian symmetry of matrices $\mathbf{C}_{ij}$ and Eqs. (24) and (25), it can be easily found that the number of needed measurements is equal to $2K(2K+1)$.

5. Experiments and results

To illustrate the justifiability and efficiency of the proposed technique, we determined the coherent-mode structure of some electromagnetic source whose cross-spectral density matrix can be measured directly in experiment and hence be known *a priori*. Such a source has been generated by means of partial destructing the coherence of linearly polarized laser radiation using a rather simple technique reported by us recently in [13] and sketched in Fig. 2.

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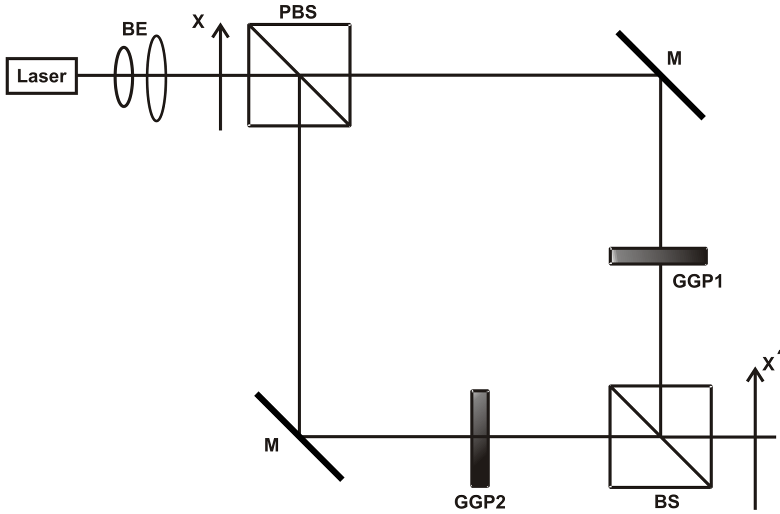

Fig. 2. Schematic illustration of the technique for generating the partially coherent and partially polarized electromagnetic source: BE, beam expander; BS, beam splitter; PBS, polarizing beam splitter; M, mirror; GGP, rotating ground glass plate.

The complex amplitude transmittance of each rotating ground glass plate is assumed being described by the function

$$t_{x(y)}(\mathbf{x}) = \exp\left[i\phi_{x(y)}(\mathbf{x})\right], \tag{26}$$

where $\phi_x(\mathbf{x})$ and $\phi_y(\mathbf{x})$ are the real random processes which obey Gaussian statistics with zero mean and second-order correlation given by the expressions

$$\left\langle \phi_{x(y)}(\mathbf{x}_1)\phi_{x(y)}(\mathbf{x}_2)\right\rangle = \sigma^2 \exp\left(-\frac{(\mathbf{x}_1-\mathbf{x}_2)^2}{2\gamma_{x(y)}^2}\right), \tag{27}$$

$$\left\langle \phi_x(\mathbf{x}_1)\phi_y(\mathbf{x}_2)\right\rangle = 0, \tag{28}$$

$$\sigma = \sqrt{\left|\phi_{x(y)}(\mathbf{x})\right|^2}. \tag{29}$$

Then, considering that the polarization plane of the primary source radiation makes an angle of 45° with x direction, the cross-spectral density matrix of generated secondary source can be well approximated as follows (see [13]):

$$\mathbf{W}^{\mathrm{SS}}(\mathbf{x}_1,\mathbf{x}_2) \approx \frac{S_0}{2}\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\alpha^2}\right)\begin{bmatrix} \exp\left(-\frac{(\mathbf{x}_1-\mathbf{x}_2)^2}{2(\gamma_x/\sigma)^2}\right) & 0 \\ 0 & \exp\left(-\frac{(\mathbf{x}_1-\mathbf{x}_2)^2}{2(\gamma_y/\sigma)^2}\right) \end{bmatrix}, \tag{30}$$

where S_0 is the spectral density (intensity) at the origin of primary source and α is the effective (rms) size of this source. It must be noted that Eq. (30) describes the so-called Gaussian Schell-model source, whose coherent-mode structure is well known [3,4]. We generated this source using a He-Ne laser ($\lambda = 633\mathrm{nm}$) as a primary source and a pair of ground glass plates with diffusion angles of 10° and 30°, considering the

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parameters α and $\gamma_{x(y)}$ to be unknown. The elements of matrix $\mathbf{W}^{\mathrm{SS}}$ for different pairs of points $(x_1 = -\xi/2, y_1 = 0)$ and $(x_2 = \xi/2, y_2 = 0)$ were measured in the modified Young's experiment sketched in Fig. 3 (see [13]). The obtained measurement data were fitted by the theoretical curve in accordance with Eq. (30).

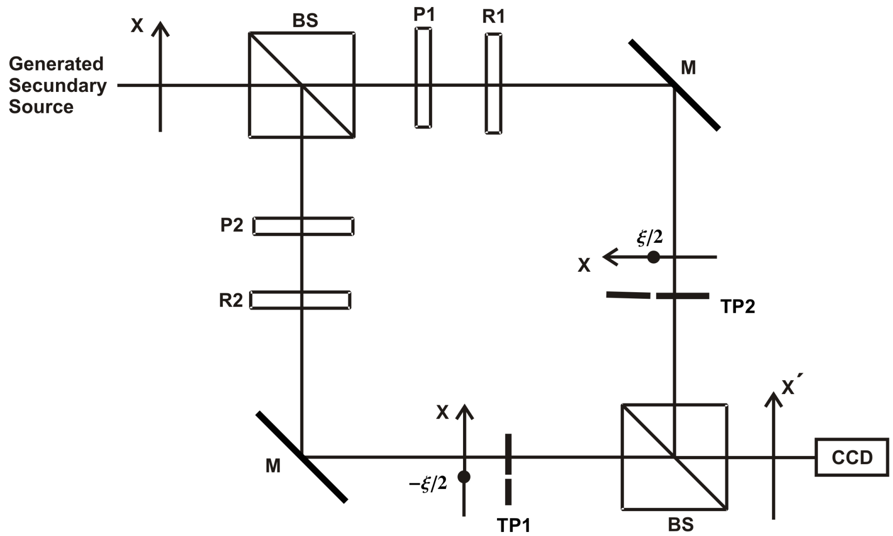

Fig. 3. Schematic illustration of modified Young experiment for measuring the cross-spectral density matrix of generated secondary source: BS, beam splitter; M, mirror; TP, translating pinhole; P, polarizer; R, polarization rotator. (The purpose of P and R is just the same as in technique sketched in Fig. 1.)

Further, employing the technique described in the previous section, we measured the coefficients $c_{ij;kl}$ for the generated source. When doing this, as the reference basis we chose the set of orthonormal Hermite-Gaussian functions

$$\psi_k(x) = \left(\frac{1}{\sqrt{\pi}\alpha 2^k k!}\right)^{1/2} \exp\left(-\frac{x^2}{2\alpha^2}\right) H_k\left(\frac{x}{\alpha}\right), \tag{31}$$

which are the actual coherent modes of a 1-D Gaussian Schell-model source [3,4]. To simplify our experiment, we truncated the reference basis by $K = 5$ terms. To encode the reference basis functions in accordance with Eq. (20), we employed two identical computer-controlled liquid-crystal spatial light modulators LC2002 from HoloEye Photonics AG, providing the amplitude-only operating mode with appropriate adjustment of polarization axes and special gamma-correction of the control signal [14]. The control video signals were generated in PC using Matlab software routines and then displayed by turns onto the liquid-crystal screen with accuracy of 256 gray levels and resolution 800×600 pixels. To provide the reliability of measurements, we undertook a special preliminary joint calibration of the amplitude transmittances in both arms of the interferometer (Fig. 1). For this purpose we made efforts to attain the minimum level (≈ 0) of the signal registered at the output of optical system applying to the spatial light modulators two orthogonal control signals and illuminating them with the same completely coherent uniform field.

The realized measurements showed that the coefficients $c_{ij;kl}$ with $i \neq j$ were almost zero while the coefficients $c_{ij;kl}$ with $i = j$ had non-zero real values, a fact that could be expected due to the nature of theoretical model given by Eq. (30). This circumstance allowed us to replace the eigenvalue problem presented by Eq. (18) by two independent eigenvalues

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problems for sub-matrices $\mathbf{C}_{xx}$ and $\mathbf{C}_{yy}$. To solve these problems, we used standard Matlab program. Once the eigenvalues have been found we computed the eigenfunctions $\varphi_{i;n}(x)$ in accordance with Eq. (19) and then the cross-spectral densities $W_{ii}(x_1, x_2)$ in accordance with Eq. (5). The results of computation are presented in Fig. 4 by solid curves. For comparison the results of direct measurements are shown by dotted curves. A slight mismatch of these curves (less than 5%) is due to an inevitable measurement error.

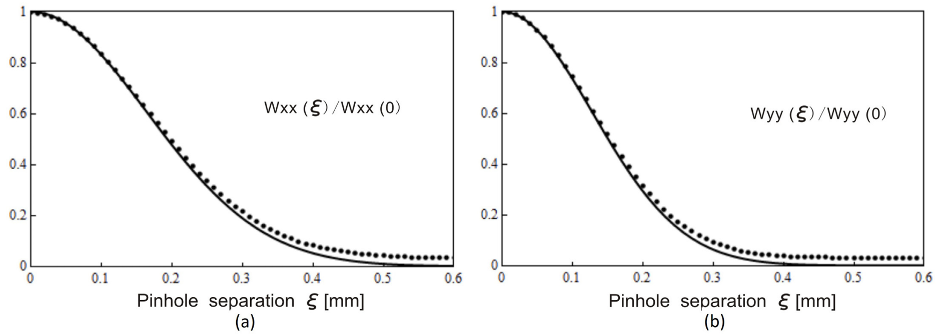

Fig. 4. Normalized cross-spectral densities of generated secondary source measured in experiment (dotted curves) and determined in accordance with the proposed technique (solid curves) for ground glass plates with diffusion angles of 10° (a) and 30° (b).

6. Conclusions

We have proposed a technique of experimental determining the coherent-mode structure of electromagnetic field. This technique is based on the coherence measurements of the field in some reference basis and represents a nontrivial vector generalization of the dual-mode field correlation method recently reported by F. Ferreira and M. Belsley for a scalar case [11]. Of course the proposed technique needs more physical and computational effort, but it is the price of vector generalization. The justifiability and efficiency of the proposed technique has been demonstrated with an example of determining the coherent-mode structure of some specially generated and experimentally characterized secondary electromagnetic source.

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