

# Sensitivity optimization of the one beam Z-scan technique and a Z-scan technique immune to nonlinear absorption

José A. Dávila Pintle,<sup>1,\*</sup> Edmundo Reynoso Lara,<sup>1</sup> and Marcelo D. Iturbe Castillo<sup>2</sup>

<sup>1</sup>Benemérita Universidad Autónoma de Puebla, Facultad de Ciencias de la Electrónica  
Av. San Claudio y 18 Sur. Col San Manuel, Puebla, Puebla. 72570, México

<sup>2</sup>Instituto Nacional de Astrofísica, Óptica y Electrónica, Luis Enrique Erro  
Tonantzintla. 72840, México

\*[jpintle@ece.buap.mx](mailto:jpintle@ece.buap.mx)

**Abstract:** It is presented a criteria for selecting the optimum aperture radius for the one beam Z-scan technique (OBZT), based on the analysis of the transmittance of the aperture. It is also presented a modification to the OBZT by directly measuring the beam radius in the far field with a rotating disk, which allows to determine simultaneously the non-linear absorptive coefficient and non-linear refractive index, much less sensitive to wave front distortions caused by inhomogeneities of the sample with a negligible loss of signal to noise ratio. It is demonstrated its equivalence to the OBZT.

© 2013 Optical Society of America

**OCIS codes:** (190.0190) Nonlinear optics; (190.4720) Optical nonlinearities of condensed matter.

---

## References and links

1. M. Sheik-Bahae, A. A. Said, T.-H. Wei, D. J. Hagan, and E. W. Van Stryland, "Sensitive measurement of optical nonlinearities using a single beam," *IEEE J. Quant. Electron.* **26**(4), 760-769 (1990).
2. J. Wang, B. Gu, Y. M. Xu, and H. T. Wang, "Enhanced sensitivity of Z-scan technique by use of flat-topped beam," *Appl. Phys. B* **95**(4), 773778 (2009).
3. T. Xia, D. J. Hagan, M. Sheik Bahae, and E. Van Stryland, "Eclipsing Z-scan measurement of  $\lambda/10^4$  wave-front distortion," *Opt. Lett.* **19**(5), 317-319 (1994).
4. G. Tsigaridas, M. Fakis, I. Polyzos, P. Persephonis, and V. Giannetas, "Z-scan technique through beam radius measurements," *Appl. Phys. B* **76**(1), 83-86 (2003).
5. P. B. Chapple, J. Staromlynska, J. A. Hermann, T. J. McKay, and R. G. McDuff, "Single-beam Z-scan: Measurement techniques and analysis," *J. Nonlinear Opt. Phys. Mater* **6**(3), 251-293 (1997).
6. I. A. Rysansky and B. Palpant, "Theoretical Investigation of the off-axis z-scan technique for nonlinear optical refraction measurement," *Appl. Opt.* **45**(12), 2773-2776 (2006).
7. R. L. McCally, "Measurement of Gaussian beam parameters," *Appl. Opt.* **23**(14), 2227-2227 (1984).
8. P. B. Chapple, "Beam waist and M2 measurement using a finite slit," *Opt. Eng.* **33**(7), 2461-2466 (1994).
9. P. J. Shayler, "Laser beam distribution in the focal region," *Appl. Opt.* **17**(17), 2673-2674 (1978).
10. S. Nemoto, "Determination of waist parameters of a Gaussian beam," *Appl. Opt.* **21**(21), 3859-3863 (1986).
11. J. M. Khosrofi and B. A. Garetz, "Measurement of a Gaussian laser beam diameter through the direct inversion of knife-edge data," *Appl. Opt.* **22**(21), 3406-3410 (1983).
12. J. A. Arnaud, W. M. Hubbard, G. D. Mandeville, B. delaClavière, E. A. Franke, and J. M. Franke, "Technique for fast measurement of Gaussian laser beam parameters," *Appl. Opt.* **10**(12), 2775-2776 (1971).
13. Y. Suzuki, and A. Tachibana, "Measurement of the  $\mu\text{m}$  sized radius of Gaussian laser beam using the scanning knife-edge," *Appl. Opt.* **14**(12), 2809-2810 (1975).
14. A. Nag, A. Kumar De, and D. Goswami, "Two-photon cross-section measurements using an optical chopper: z-scan and two-photon fluorescence schemes," *J. Phys. B: At. Mol. Opt. Phys.* **42**(6), 065103, (2009).

15. I. Bhattacharyya, S. Priyadarshi, and D. Goswami, "Molecular structure-property correlations from optical non-linearity and thermal-relaxation dynamics," *Chem. Phys. Lett.* **469**, 104-109, (2009).
  16. A. Rogalski and Z. Bielecki, "Detection of optical radiation," *Bull. Pol. Ac.: Tech.* **52**(1), 43-66 (2004).
  17. C. D. Motchenbacher and J. A. Connelly, *Low-noise Electronic System Design* (John Wiley & Sons Inc, 1993).
  18. J. Phillips and K. Kundert, "Noise in mixers, oscillators, samplers, and logic an introduction to cyclostationary noise," *Proceedings of the IEEE custom integrated circuits conference*, 431-439, (2000).
  19. B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (John Wiley & Sons Inc, 1991).
  20. X. Liu, S. Guo, H. Wang, N. Ming, and L. Hou, "Investigation of the influence of finite aperture size on the Z-scan transmittance curve," *J. Nonlinear Opt. Phys. Mater.* **10**(4), 431-439 (2001).
- 

## 1. Introduction

Since the publication of the one beam Z-scan technique (OBZT) by Bahae-Sheik et al. in 1989 [1], it has become very popular for determining nonlinear absorptive coefficient and refractive index of a thin sample, because its simplicity and sensitivity. The nonlinear refractive index is determined from the transmittance of the sample measured through an aperture in the far field as the sample is displaced around the focal position of a Gaussian beam propagating along  $z$  axis.

However, the aperture introduces important disadvantages, namely, it must be aligned with the axis of the beam ( $z$  axis) to measure on axis transmitted power ( $P$ ), second,  $P$  is only a small fraction of the total laser power ( $P_0$ ), compromising the signal to noise ratio (SNR), and perhaps the most important of all, when the sample exhibits nonlinear absorption, there is no way to know whether changes in  $P$  due to changes in the refractive index or absorption when the sample is displaced, forcing to repeat the experiment without the aperture for measuring the nonlinear absorption and deduct it, as will be seen later.

Since then, several modifications have been proposed to improve the sensitivity of the OBZT, namely, modifying the beam profile to enhance the aperture transmittance [2], substituting the aperture [3] or eliminating the aperture [4] where the beam width is directly measured using a laser beam profiler based on CCD camera.

Although the effect of the aperture has been studied for Gaussian beams [5, 6] little attention has been paid to the selection of the aperture radius for optimizing the signal to noise ratio of the OBZT.

The present paper presents a criteria for selecting the optimum aperture radius, based on the analysis of the physical variable of the OBZT, it is presented also a Z-scan technique based on a direct measurement of the beam width in the far field, fast and cheap, it is demonstrated its equivalence, the signal to noise ratio of the OBZT with the optimum aperture radius is compared with the technique presented, such technique allows to measure simultaneously the nonlinear absorption and refractive index.

## 2. Description of the chopper-width technique

Many techniques have been developed for measuring the beam width of a Gaussian beam, such as the slit scan technique [7, 8], the pinhole technique [9], but among all, the most extended is the knife-edge technique [10, 11], because is an inexpensive, and accurate beam profiler. In this technique, a knife located at ( $\zeta$ ) eclipses the beam transversely to its propagation axis ( $z$  axis), the beam width ( $W(\zeta)$ ) is determined from the derivative plot of the transmitted power versus the knife-edge position. This method is simple but a bit tedious, slow and noise sensitive. These issues can be overcome with the technique described below.

The chopper-width measurement technique basically consists in to extract  $W(\zeta)$ , from the rise or fall time, of a electric signal [12] generated by switch- on and switch-off the beam light, with a slotted rotating disk (named chopper) at the position ( $\zeta$ ) [13]. This technique is alike to the knife-edge technique but faster, allowing real-time measurements. The width

$W(\zeta)$  measured in this technique is larger than the obtained with the knife-edge technique, due the curved trajectory to cut the beam, rather than perpendicular. This drawback can be overcome calibrating the chopper-width technique, this is presented in the appendix at the end of this work. Let us consider a Gaussian beam propagating along the  $z$  axis impinging on a photoreceptor, the chopper is located at the  $\zeta$  position where the  $W(\zeta)$  is to be measured, like the chopper has  $l$  slots and rotates with constant angular frequency  $\Omega_0$ , as shown in Fig. 1. To each blade will take  $\tau = \frac{\theta}{\Omega_0}$  seconds to sweep the angle  $\theta$  subtended by the beam. If  $R \gg W(\zeta)$ ,  $\tau$  is given by

$$\tau \cong \frac{2W(\zeta)}{\Omega_0 R} \quad (1)$$

where  $R$  is the distance from the axis of rotation to the beam axis, therefore, Eq. (1) provides a way to measure  $W(\zeta)$  from the time  $\tau$ .

Then, it is proposed to use the width-measurement technique, based on Eq. (1), to obtain Z-scan curves (we shall name chopper Z-scan technique) like Tsigaridas [4], but in a simpler and inexpensive way. It is important to mention that until now, choppers have been used in the OBZT as modulation instruments [14, 15], in order to improve the detection of  $P$ , however, here the chopper is used as meter width. Before experimentally demonstrate the feasibility and advantages of this approach, let us present its justification and the comparison of the sensitivities of chopper and OBZT.

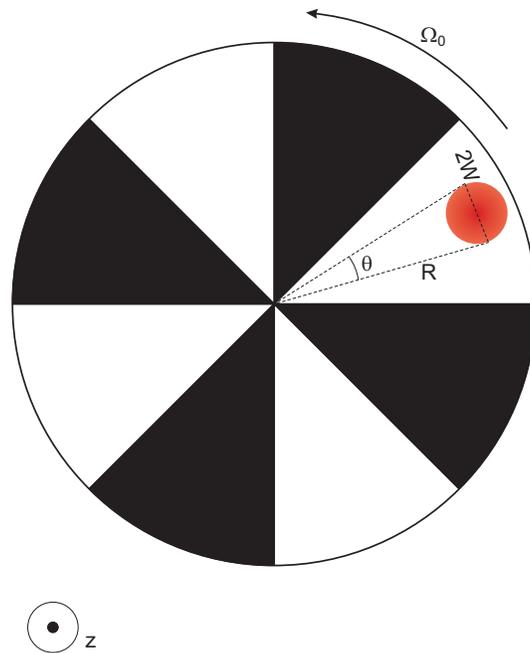


Fig. 1. Front view of a slotted disk which rotates at a frequency  $\Omega_0$  and eclipses a Gaussian beam shown in red.  $R$  is the distance from the rotation axis to the center of the beam,  $W$  is the beam width, and  $\theta$  is the angle subtended by the spot beam.

### 3. Equivalence and sensitivities

In the OBZT (on axis measurement) for CW radiation, the normalized transmittance ( $T(z)$ ) of the aperture located at  $\zeta$  fixed, is defined as [1]

$$T(z) = \frac{P(W(z), \rho_0)}{P(W(\infty), \rho_0)} \quad (2)$$

where  $z$  is the position of the nonlinear sample referred to the minimum waist position ( $z = 0$ ) and the power transmitted by the aperture is given by

$$P(W(z), \rho_0) = P_0 \left( 1 - \exp \left( -\frac{2\rho_0^2}{W(z)^2} \right) \right) \quad (3)$$

$\rho_0$  is the aperture radius,  $W(z)$  and  $W(\infty) = W_L$  are the beam radius at the position of the aperture when the sample is located at  $z$  and far away ( $\infty$ ) from the focus (linear regime) respectively,  $P_0$  is the optical power of the beam incident to the aperture. Henceforth, we shall assume the implicit dependence of  $W$  on  $z$  in order to simplify the notation.

Taking into account that the reference is the unity, i.e.  $T(\infty) = 1$ , then, the effect of the presence of sample is measured from this value. Also, it is easy to realize that  $\frac{\Delta T(z)}{T(\infty)}$  represents the relative change of the transmitted power in the linear regime, i.e.  $\frac{\Delta P}{P(W_L, \rho_0)}$ , also this fact allows to establish the following relation

$$\frac{\Delta T(z)}{T(\infty)} = \frac{\Delta P}{P_0} S^{-1} \quad (4)$$

where the relative optical change and linear transmittance are respectively defined as

$$\frac{\Delta P}{P_0} = \frac{P(W, \rho_0) - P(W_L, \rho_0)}{P_0} \quad (5)$$

and

$$S = 1 - \exp \left( -\frac{2\rho_0^2}{W_L^2} \right) \quad (6)$$

The power transmitted (Eq. (3)) is what physically is measured (physical variable), hence, the sensitivity of Z-scan technique is determined by the minimum value of this quantity can be discerned (or discriminated) due to the presence of noise in the measurement.

#### 3.1. The signal of the OBZT

In a Z-scan experiment due to the presence of the nonlinear sample, the beam width changes from  $W_L$  to  $W$ , therefore, the beam width change ( $\Delta W$ ) is

$$\Delta W = W - W_L \quad (7)$$

this causes that the transmitted power ( $P$ ), changes by  $\Delta P$ , let us calculate the optimum value of  $\rho_0$  to detect ( $\frac{\Delta P}{\Delta W}$ ), the equation to be solved is

$$\frac{\partial^2}{\partial \rho_0 \partial W} P(W, \rho_0) = 0 \quad (8)$$

which gives the following result

$$\rho_0 = \frac{W_L}{\sqrt{2}} \quad (9)$$

i.e., when the aperture radius is 70.7% of  $W_L$  (using the criteria of  $\frac{1}{e^2}$  for measuring the beam width), it is achieved the highest sensitivity for detecting changes in  $P$ . To reinforce this assertion, in Fig. 2, it is shown an experimental plot of  $\Delta P$  as function of the ratio  $\frac{\rho_0}{W_L}$ , as can be seen in such plot the maximum sensibility effectively occurs when  $\rho_0 \cong 0.71W_L$ , corroborating Eq. (9), the experimental details are given in section 4.

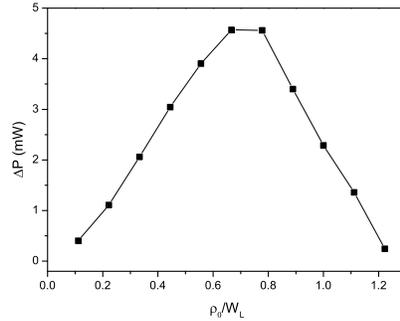


Fig. 2. Sensitivity of an aperture of radius ( $\rho_0$ ) to detect changes in its transmitted power ( $\Delta P = P(W, \rho_0) - P(W_L, \rho_0)$ ), when the incident beam width changes from ( $W_L \rightarrow W$ ).

For this optimum aperture, Eq. (5) gives the following result

$$\frac{\Delta P}{P_0} = \exp(-1) - \exp\left(-\frac{W_L^2}{W^2}\right) \quad (10)$$

for small changes ( $\Delta W \ll W_L$ ), Eq. (10) can be approximated to

$$\frac{\Delta P}{P_0} \cong 2\exp(-1) \left(\frac{-\Delta W}{W_L}\right) \quad (11)$$

Eq. (11) is important because allows to relate beam width changes ( $\Delta W$ ) with changes in the transmitted power (Eq. (4)). Substituting Eq. (6), Eq. (9) and Eq. (11) into Eq. (4) gives

$$\frac{\Delta T(z)}{T(\infty)} \cong 1.16 \left(\frac{-\Delta W}{W_L}\right) \quad (12)$$

Eq. (12) provides the equivalence of the chopper technique. The minus sign in Eq. (12), means that the chopper Z-scan curve is inverted with respect to the OBZT Z-scan, because when there is a positive increment of beam width ( $\Delta W > 0$ ) the beam intensity and therefore the aperture transmittance diminishes ( $\Delta T < 0$ ). In the following we shall work with  $\frac{\Delta W}{W_L}$  instead of  $\frac{\Delta T(z)}{T(\infty)}$ .

To conclude this subsection, using Eq. (12), substituting Eq. (9) into Eq. (6) and using the following result [1] (assuming the same approximations, i.e. an incident circular Gaussian beam, small incident power, small phase changes, and thin sample.)

$$\Delta T_{p-v} \simeq 0.406(1 - S)^{0.25} |\Delta \Phi_0| \quad (13)$$

the changes on-axis phase shift at the focus ( $\Delta \Phi_0$ ) can be related to the beam width changes, as follows:

$$\frac{\Delta W_{p-v}}{W_L} \simeq 0.273 |\Delta \Phi_0| \quad (14)$$

where  $\Delta W_{p-v}$  is the difference peak-valley of the chopper Z-scan curve.

### 3.2. The signal for the chopper Z-scan

For the chopper Z-scan technique, the elapsed time  $\tau$  for eclipsing the beam width ( $W$ ) is given by Eq. (1), when  $W$  changes due to the presence of the sample,  $\tau$  also changes by  $\Delta\tau$ , given by:

$$\Delta\tau = \frac{2}{\Omega_0 R} \Delta W \quad (15)$$

therefore, the minimum  $\Delta\tau$  that noise allows to discriminate, determines, the minimum  $\Delta W$  that can be resolved with this technique.

### 3.3. Calculus of noise power for both techniques

As mentioned before, the optical power is the physical variable to be measured for the OBZT through a photoreceptor system, it usually consists of a photodiode and a trans-impedance amplifier (see Fig. 3(a)), for this system the voltage  $V_{ph}$  (which only depends on the sample position  $z$ ) is:

$$V_{ph} = R_L \mathfrak{R} P \quad (16)$$

$R_L$  is the amplifier zero frequency gain in Ohms,  $\mathfrak{R}$  is the responsivity of the photodiode in  $\frac{\text{Ampere}}{\text{Watt}}$  and  $P$  is the optical power incident to the photodiode in *Watt*.

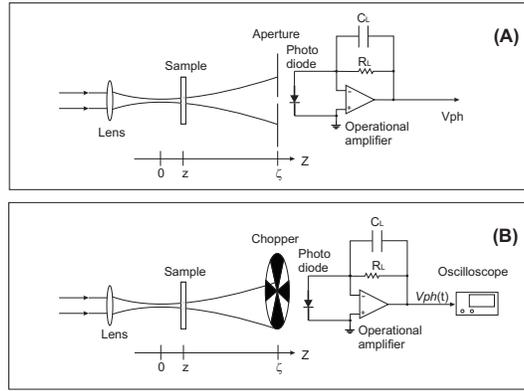


Fig. 3. A) One beam Z-scan set up, B) Chopper Z-scan set up.

In the case of small  $P$ , the photoreceptor is limited by thermal noise [16], therefore, the signal to noise ratio ( $\frac{S}{N}$ ) in this regime is

$$\frac{S}{N} = \frac{R_L (\mathfrak{R} P)}{\sqrt{4kTR_L \Delta f}} \quad (17)$$

where  $k$  is the Boltzmann constant,  $T$  is the temperature in Kelvin and  $\Delta f$  is the detection bandwidth of the photoreceptor in *Hz* given by [17]

$$\Delta f = \frac{1.57}{2\pi R_L C_L} \quad (18)$$

the expression in parentheses in Eq. (17) is the photo-current generated by the photodiode.

The minimum signal detectable occurs when ( $\frac{S}{N} = 1$ ), also, assuming stationary noise sources and non-memory measurements, then, the minimum change ( $\Delta P_{\min}$ ) detectable is

$$\Delta P_{\min} = 2 \frac{\sqrt{4kTR_L \Delta f}}{R_L \mathfrak{R}} \quad (19)$$

to obtain an expression for the magnitude of the minimum change of width detectable ( $\Delta W_{\min}^{OB}$  the superscript comes from One Beam), Eq. (19) is substituted into Eq. (11)

$$\left| \frac{\Delta W_{\min}^{OB}}{W_L} \right| = \exp(1) \frac{\sqrt{4kTR_L\Delta f}}{R_L\Re P_0} \quad (20)$$

the expression under the square root symbol, represents the thermal noise power ( $P_n$ ), of course, if other additive noise sources were present,  $P_n$  would represent the total noise power, hence,

$$\left| \frac{\Delta W_{\min}^{OB}}{W_L} \right| = \frac{2.72\sqrt{P_n}}{R_L\Re P_0} \quad (21)$$

the sensitivity of the OBZT is given by Eq. (21).

For the chopper Z-scan, the total laser power ( $P_0$ ) impinges on the photodiode, therefore, a lower gain value of  $R_L$  is needed, lowering the noise power due to this cause, however, the bandwidth requirement ( $\Delta f$ ) is greater than the OBZT, somewhat compensating the previous advantage, as shown in the following subsection.

### 3.3.1. Bandwidth need of the chopper z-scan

For the chopper technique, the incident optical power impinging on the photodiode  $P_{ch}(t)$  is a periodic function of the time  $t$ , due to periodic eclipsing;  $P_{ch}(t)$  is given by:

$$P_{ch}(t) = \frac{P_0}{2} \left( 1 - \operatorname{erf} \left( \sqrt{2} \frac{R \sin(\Omega_0 t)}{W} \right) \right) \quad (22)$$

let us define the following ratio

$$\beta = \frac{R}{W} \quad (23)$$

for  $\Omega_0 t \ll 1$  the raising part of Eq. (22) can be approximated to

$$P_{ch}(t) \cong \frac{P_0}{2} \left( 1 - \operatorname{erf} \left( \sqrt{2} \beta \Omega_0 t \right) \right) \quad (24)$$

the Fourier series of  $P_{ch}(t)$  using the approximation (Eq. (24)) is

$$P_{ch}(t) = \frac{1}{2} - \frac{2}{\pi} \sum_{m=0}^{\infty} \frac{1}{2m+1} \exp \left( -\frac{1}{8} \left( \frac{2m+1}{\beta} \right)^2 \right) \sin((2m+1)\Omega_0 t) \quad (25)$$

if the series is cut up to  $m \geq 2\beta$  ( $m$  integer) harmonic, the error is less than 0.6%, therefore the chopper Z-scan bandwidth need ( $\Delta f_c$ ) in Hz is:

$$\Delta f_c = 4\beta f_0 \quad (26)$$

where it has been used the fact that  $\Omega_0 = 2\pi f_0$  and  $\Omega_0 t \ll 1$ , hence, the photoreceptor must satisfy the following requirement

$$\Delta f_c \leq \Delta f \quad (27)$$

where  $\Delta f$  is given in Eq. (18).

The Eq. (26) can be approximately obtained under the following reasoning: the eclipsing time  $\tau$  was calculated in Eq. (1), therefore, the photoreceptor must at least be as fast as the eclipsing time, consequently, its bandwidth  $\Delta f_c$  must be

$$\Delta f_c \geq \frac{1}{\tau} = \frac{\Omega_0 \beta}{2} \quad (28)$$

the following is also meet  $\Omega_0 = 2\pi f_0$  where  $f_0$  is the rotation frequency in Hz, thus, Eq. (28) can be expressed as

$$\Delta f \geq \pi \beta f_0 \quad (29)$$

### 3.3.2. Minimum change of width detectable with the chopper Z-scan technique

The eclipsing time  $\tau$  is the time elapsed for  $V_{ph}(t)$  to vary from  $V_{th1}$  to  $V_{th2}$  as shown in Fig. 4, therefore,  $V_{ph}(t)$  is compared with the threshold voltages  $V_{th1}$  and  $V_{th2}$ , hence,  $V_{ph}(t)$  fluctuations due to noise ( $n(t)$ ) result in uncertainty in the comparison process (*jitter*), in this case the following relation is satisfied between the variance in the metric  $j$  of the time of the threshold crossing [18]

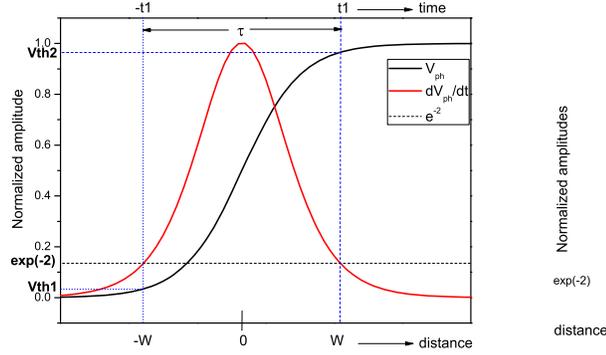


Fig. 4. Establishment of the threshold voltages  $V_{th1}$  and  $V_{th2}$  using  $e^{-2}$  criteria for measuring the beam width  $W$  from  $V_{ph}(t)$  (black line) and the beam profile (red line).

$$\text{var}(j(t_c)) \cong \frac{\text{var}(n(t_c))}{\left(\frac{dV_{ph}(t_c)}{dt}\right)^2} \quad (30)$$

where  $t_c$  is the expected time of the threshold crossing and  $V_{ph}(t)$  is the voltage delivered by the photoreceptor system. In order to compare both techniques let us assume that the same amplifier than the OBZT is used and Eq. (27) is satisfied, then,

$$V_{ph}(t) \cong R_L \mathfrak{R}P_{ch}(t) \quad (31)$$

$P_{ch}(t)$  is given by Eq. (22). For noise with zero mean,  $\text{var}(n(t)) = P_n$  [19],  $P_n$  is the noise power of  $n(t)$ , therefore, equation (30) can be expressed as

$$\text{var}(j(t_c)) \cong \frac{P_n}{\left(\frac{dV_{ph}(t_c)}{dt}\right)^2} \quad (32)$$

from Eq. (1) and Eq. (32) we can establish the following relation

$$\sqrt{\text{var}(W)} \cong \frac{\Omega_0 R}{2} \frac{\sqrt{P_n}}{\left|\frac{dV_{ph}(t_c)}{dt}\right|} \quad (33)$$

Note that  $\frac{dV_{ph}(t)}{dt}$  is proportional to the Gaussian profile, referring to Eq. (33) and Fig. 4 the uncertainty in measuring  $\Delta W$  is minimized if  $V_{th1}$  and  $V_{th2}$  are set as close as possible to the maximum value of the profile, hence, if they are established where the profile drops to  $\pm \frac{W_L}{3}$

(i.e.  $V_{th1} \cong 0.25V_{ph}(t)$  and  $V_{th2} \cong 0.75V_{ph}(t)$ ), then from Eq. (1)

$$t_c = \pm \frac{2W_L}{3\Omega_0 R} \quad (34)$$

using Eq. (31), Eq. (24) and Eq. (34) the derivative of  $V_{ph}$  is

$$\left| \frac{d}{dt} V_{ph}(\pm t_c) \right| = \sqrt{\frac{2}{\pi}} e^{-\frac{8}{9}} \beta \Omega_0 (R_L \Re P_0) \quad (35)$$

substituting Eq. (35) into Eq. (33) and taking into account that two comparisons are done (at  $V_{th1}$  and  $V_{th2}$ ), the minimum relative change of the beam width ( $W_{min}^{Ch}$  the superscript comes from Chopper) that this technique can resolve is

$$\frac{\Delta W_{min}^{Ch}}{W_L} = 3 \frac{\sqrt{P_n}}{R_L \Re P_0} \quad (36)$$

comparing Eq. (36) with Eq. (21) under the same power of noise both techniques are almost equally sensitive, however, the chopper technique does not require an aperture, eliminating the need to align it with the beam axis and due to its larger bandwidth, a measurement is carried out in fraction of second, allowing to perform an average over a much larger sample under the same observation time, and perhaps the most important feature it is absorption immune.

Before turning to the experimental results it should be noted, that for practical purposes, due to the physical variable for the chopper technique is time, then, according to Eq. (1), the relative changes in the width are equal to the relative changes in the rising time (i.e.  $\frac{\Delta \tau}{\tau_L} = \frac{\Delta W}{W_L}$ ), therefore it can be obtained a Z-scan curve plotting  $\tau$  versus  $z$ , and to use Eq. (37) derived from Eq. (1) and Eq. (13) to estimate the phase change

$$\frac{\Delta \tau_{p-v}}{\tau_L} \simeq 0.273 |\Delta \Phi_0| \quad (37)$$

where  $\Delta \tau_{p-v}$  is the peak-valley difference of the Z-scan curve, and  $\tau_L$  is the rising time of  $V_{ph}(t)$  in the linear regime.

#### 4. Experimental results

Equation (9) was verified using a single mode laser JDS 1145P 30mW HeNe, a positive lens with 3.5cm focal length, an adjustable aperture, a power meter Thorlabs PM100 disposed according to Fig. 5 under the following reasoning: Without the sample, the power of the light ( $P$ )

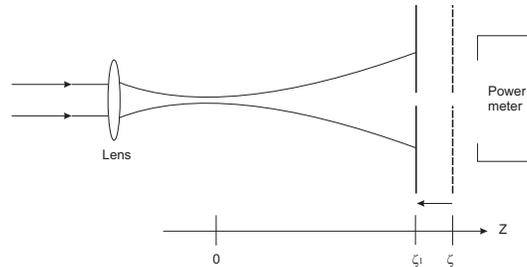


Fig. 5. Set up for determining the optimum aperture radius for measuring transmitted power changes.

passing through the aperture is function of its position ( $\zeta$ ) along beam axis and its radius ( $\rho_0$ ),

i.e.  $P(W(\zeta), \rho_0)$ , thus, the aperture was located at a distance ( $\zeta$ ) where the beam width was ( $W(\zeta) = 4.5\text{mm}$ ); then  $P(W(\zeta), \rho_0)$  was measured for ten increasing radius  $\rho_0$ , after, the aperture was displaced at ( $\zeta_1$ ) where the width was ( $W(\zeta_1) = 4\text{mm}$ ),  $P(W(\zeta_1), \rho_0)$  was measured for the same values of  $\rho_0$  as before. In Fig. 2 it is shown a plot of  $\Delta P = P(W(\zeta_1), \rho_0) - P(W(\zeta), \rho_0)$  versus  $\frac{\rho_0}{W_L}$  where it can be seen that the optimum  $\rho_0$  for detecting changes in the transmitted power is  $\frac{W(\zeta)}{\sqrt{(2)}}$ .

To carry out the OBZT, it was used a bacteriorhodopsin film  $8\mu\text{m}$  thick, the JDS 1145P laser, a 5 cm lens, a Thorlabs PIN photodiode SM1PD1A, a home-made trans-resistance amplifier with  $R_L = 82\text{k}\Omega$ ,  $C_L = 10\text{nF}$  and operational amplifier OP27, an oscilloscope model TDS1012C-EDU for monitoring  $V_{ph}$ , disposed according to the setup shown in Fig. 3(A), the distance from the output laser to the lens was 50cm, the distance of the lens to the aperture was 50cm; to perform the chopper technique the same elements were used arranged according to Fig. 3(B), replacing the aperture with a disk with 10 slots rotating at  $\Omega_0 = 18.85 \frac{\text{rad}}{\text{s}}$  (chopper Thorlabs model MC1000A).

#### 4.1. Discussion of results

For both techniques in order to minimize disruption to the sample, the laser power was attenuated up to  $40\mu\text{W}$ .

##### 4.1.1. OBZT

For nonlinear absorptive samples, the OBZT curve is a mixture of nonlinear refractive and absorptive contributions [20], therefore, in order to obtain a free absorption Z-scan curve, it is proceeded as follows: A Z-scan experiment is performed without the aperture, the normalized curve obtained is shown in the inset of Fig. 6, subsequently, a second Z-scan experiment is done with the aperture, at the same positions as the inset of Fig. 6, the values measured are divided by their corresponding values of the curve in the inset; in Fig. 6 it is shown the free nonlinear absorption Z-scan curves for different aperture radii, where  $\Delta V = V_{ph} - V_L$ ,  $V_L$  is the voltage  $V_{ph}$  measured when the sample is far from the focus (linear regime). Again, the optimum radius turned out to be determined by the Eq. (9).

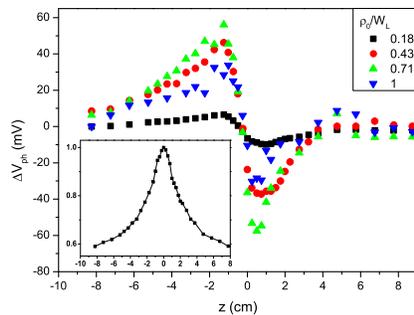


Fig. 6. Z-scan curves free of nonlinear absorption for a bacteriorhodopsin sample, using different aperture radii ( $\rho_0$ ),  $W_L = 3\text{mm}$ , the laser power was  $P_0 = 40\mu\text{W}$ . In the inset is shown the normalized  $V_{ph}$  measured without aperture, useful to obtain the free nonlinear absorption curves.

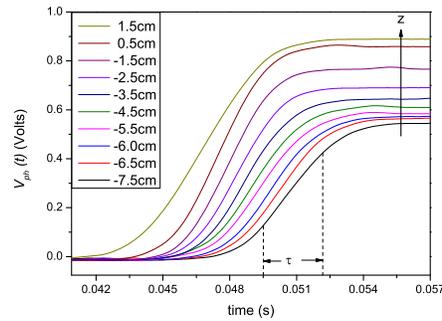


Fig. 7. Rising part of  $V_{ph}(t)$ , notice the change of amplitude and slope as function of the sample position, the arrow indicates the increasing sample position  $z$ .

#### 4.1.2. Chopper Z-scan

In Fig. 7 it is shown the raising part of  $V_{ph}(t)$  recorded with the oscilloscope, for illustrative purposes different curves were displaced to the left as the position of the sample was increased, from such figure it is clear how the chopper technique works: from the amplitude of  $V_{ph}(t)$  the nonlinear absorption is estimated and from the raising time ( $\tau$ ) the refractive index is estimated because  $\tau$  depends only on the beam width (see Fig. 4).

In Fig. 8 it is shown  $\tau$  vs  $z$  (the position of the sample), also in the inset is shown the amplitude of the pulses (such curve represents the transmittance).

From Fig. 8, it is demonstrated the capacity of chopper technique to estimate both nonlinear absorption and refractive index in one experiment.

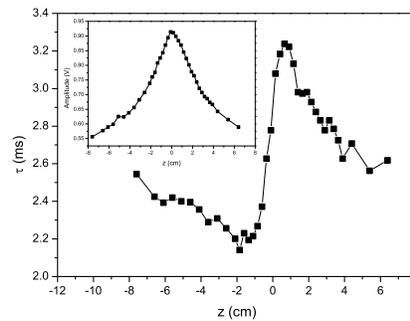


Fig. 8. Rising time ( $\tau$ ) of  $V_{ph}(t)$  as function of the sample position ( $z$ ), in the inset it is shown the amplitude of  $V_{ph}(t)$  versus  $z$  (which represents the nonlinear transmittance of the sample).

In order to highlight the advantages of the chopper technique, in Fig. 9 are presented the Z-scan curves of the relative changes of the physical variables obtained with the chopper and OBZT techniques, for the OBZT the ratio  $\frac{P_0}{W_L}$  was adjusted as small as our iris allowed ( $\frac{P_0}{W_L} = 0.1$  in the small aperture limit [5] in order to minimize the nonlinear absorption effects), the laser power was attenuated up to  $10\mu W$ ; from Fig. 9 it is evident that the nonlinear absorption distorts the OBZT Z-scan curve [20] even with this aperture radius, unlike the chopper technique.

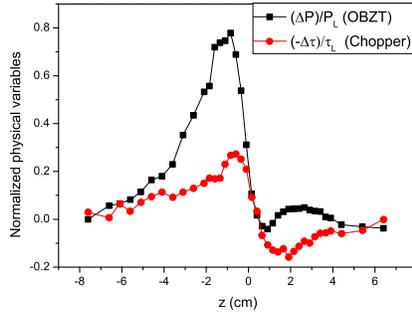


Fig. 9. Z-scan curves of the physical variables of the two techniques when  $P_0 = 10\mu W$ , for the OBZT  $\rho_0 = 0.1W_L$  and a lock-in amplifier was needed,  $P_L$  and  $\tau_L$  are the values measured for OBZT and chopper techniques respectively when the sample was located far from the focus.

In Fig. 9, the OBZT curve is twice than the chopper curve, because of the following reason: the Eq. (11) for the case of  $\rho_0 \neq \frac{W_L}{\sqrt{2}}$  is

$$\frac{\Delta P}{P_0} \cong - \left( 4 \frac{\rho_0^2}{W_L^2} \exp \left( -2 \frac{\rho_0^2}{W_L^2} \right) \right) \frac{\Delta W}{W_L} \quad (38)$$

using  $\frac{\Delta W}{W_L} = \frac{\Delta \tau}{\tau_L}$ ,  $\frac{\rho_0}{W_L} = 0.1$  and Eq. (3) for calculating  $P(W(\infty), \rho_0) = P_L$ , the Eq. (38) gives the following result:

$$\frac{\Delta P}{P_L} = 1.98 \frac{(-\Delta \tau)}{\tau_L} \quad (39)$$

one might think that the OBZT is more sensitive than the chopper technique, however, it should not be confused for the result of Eq. (39), the optical power is fully utilized in the chopper technique, achieving a better signal to noise ratio (SNR) than the SNR of the OBZT, for example, for the opening radius  $\rho_0 = 0.1W_L$ , the optical power detected with the OBZT (Eq. (3)) technique is 50 times less than the maximum power ( $P_0$ ) detected with the chopper technique, this is the reason of the use of a device that improves the SNR (lock-in). The OBZT increases its SNR as  $\rho_0$  is increased up to  $\frac{W_L}{\sqrt{2}}$  (value at which both techniques have the same SNR), however, for this value, the contribution of the nonlinear absorption is very dominant, that it is necessary a second experiment to estimate it and deduct it, as was done for obtaining Fig. 6.

Finally, in Fig. 10 it is shown  $V_{ph}(t)$  measured when a sample distorts the beam profile, such distortion induces fast changes in  $V_{ph}(t)$ , which are filtered by the finite bandwidth of the amplifier (Eq. (18)), for this reason, it is advisable to keep the bandwidth as low as possible, i.e.

$$\Delta f \longrightarrow \Delta f_c \quad (40)$$

## 5. Conclusion

We have demonstrated that a direct measurement of the beam width through a chopper, the Z-scan technique is simplified without compromising its sensitivity, also the speed of data acquisition increases remarkably, allowing to perform averages with larger data, resulting in neater Z-scan curves.

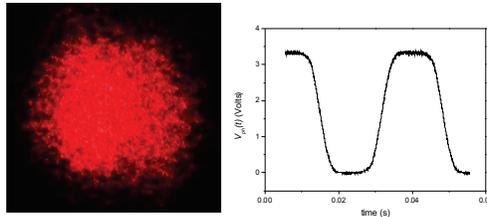


Fig. 10. Picture of a Gaussian beam profile distorted by an inhomogeneous sample, the graph shows the detected signal  $V_{ph}(t)$ , since the chopper technique works with the integral of the Gaussian profile and thanks to the limited bandwidth of the trans-impedance amplifier, the speckle affects little to the measurement.

We have established that the aperture radius plays an important role in the sensitivity and the optimum value is  $\frac{W_L}{\sqrt{(2)}}$  where  $W_L$  is the beam width (using the criteria of  $\frac{1}{e^2}$ ) in the linear regime.

For OBZT, the Z-scan curves are distorted by the presence of nonlinear absorption accentuated by increasing the radius of the aperture, forcing to correct them with the nonlinear absorption curve as can be seen in Fig. 6, whereas the proposed technique is immune to this effect.

A reliable method of measurement Gaussian beam by means of a chopper is given and confirmed experimentally as can be seen in Fig. 12.

We have found that the chopper technique is much less sensitive to wavefront distortions caused by inhomogeneities of the sample unlike OBZT which strongly depends on it.

## 6. Appendix

As we mentioned, the chopper describes a curved trajectory rather than a straight one as with the knife edge technique, this causes that the beam width measured with the chopper ( $W_{ch}$ ) is larger than that of the knife ( $W_{kn}$ ), this difference is negligible if condition ( $R \gg W$ ) is met, if not, the difference can be deducted if an expression for it is found, by measuring  $W(z)$  of a Gaussian beam, propagating along z-axis with both techniques at different positions ( $\zeta$ ) under the same experimental conditions. In Fig. 11,  $W_{kn}$  is plotted versus  $W_{ch}$ , the data fitting Eq. (41) allows to find the relation between both techniques.

$$W_{kn} = 0.8W_{ch} + 74.8\mu m \quad (41)$$

In Fig. 12 it is show the width measured of a the laser used in this paper, where it can be seen an excellent agreement between knife and calibrated chopper techniques.

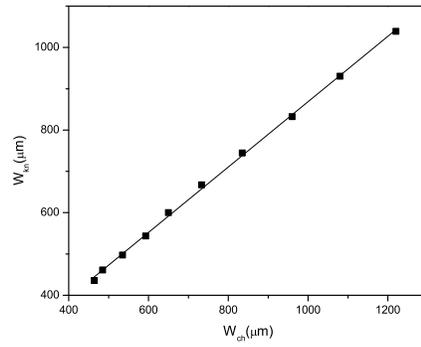


Fig. 11. Calibration of the chopper width measurement.

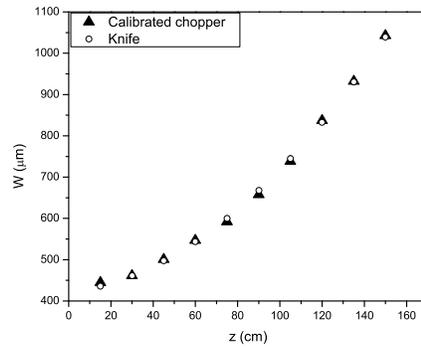


Fig. 12. Equivalence in  $W$  measurements given by the calibrated chopper and knife edge techniques.

## Acknowledgments

This work has been sponsored by CONACyT, grants 51757 and 84008.