Optical Engineering

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SPIE

Optical Engineering 52(5), 053606 (May 2013)

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Abstract. We present the validation for Ronchigram recovery with the random aberrations coefficients (ReRRCA) algorithm. This algorithm was proposed to obtain the wavefront aberrations of synthetic Ronchigrams, using only one Ronchigram without the need for polynomial fits or trapezoidal integrations. The validation is performed by simulating different types of Ronchigrams for on-axis and off-axis surfaces. In order to validate the proposed analysis, the polynomial aberration coefficients that were used to generate the simulated Ronchigrams were retrieved. Therefore, it was verified that the coefficients correspond to the retrieved ones by the algorithm. The results show that the ReRRCA algorithm retrieves the aberration coefficients from the analyzed Ronchigram with a maximum error of 9%. [DOI: 10.1117/1.OE.52.5.053606]

Subject terms: Ronchi test; aberration coefficients; genetic algorithm; images processing; lateral shear interferometry.

Paper 130170 received Feb. 1, 2013; revised manuscript received Apr. 4, 2013; accepted for publication Apr. 30, 2013; published online May 24, 2013.

1 Introduction

During the polishing process of an optical surface is important to know the evolution of the shape of the surface that is manufactured, i.e., the surface being tested. This initial surface generally has small irregularities (peaks and valleys) that, so far, can only be quantified with interferometric testing[1]; therefore, by performing such tests it is possible to know how close it is to the ideal surface to be manufactured. In almost all manufacturing processes one of the most commonly used tests in optical workshops that assess the quality of the surface is the classical Ronchi test; this is because the versatility and flexibility of this test are very broad.[1,2]

In the literature we can find several procedures for interferogram analysis by using genetic algorithms,[3–5] which describe the procedure for finding the phase of a noisy interferogram. However, the genetic algorithm is a heuristic optimization method. In this paper, we propose an alternative technique to the genetic algorithms by minimizing a defined merit function. The proposed algorithm does not have similar stages as those used in a genetic algorithm: (initial population crosses, mutation, etc.). Instead there are two main cycles, where the first cycle is heuristic due the random function used for the variation of each coefficient, while the second cycle is not heuristic because the variation of the coefficients is controlled. By these reasons we cannot directly compare the proposed algorithm with a genetic algorithm. The proposed algorithm is mainly applied to a Ronchigrams analysis, where it is only necessary to analyze one Ronchigram without the need of polynomial fit or trapezoidal integrations as in conventional Ronchigram analysis.

For optical surface metrology it is necessary to obtain accurate and reliable measurements and have a good method of analysis. The importance of this work lies in the generation of a new technique for quantitative analysis. Another aspect developed in this paper is to have a computer program for Ronchigram analysis.

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1.1 *Ronchi Test*

A Ronchi test is a powerful method used to measure aberrations in optical systems, since the surface errors can be estimated from the deviation of the observed fringe pattern on the exit pupil and along the polishing work a comparison is made against a previously calculated theoretical pattern.[1,2]

The fringes in the Ronchi test are usually studied with geometrical theory, where the fringes in the Ronchigram are the result of the ray deviations from an ideal path. This is caused by errors in the slopes of the surface being tested; see Fig. 1. From the point of view of the wave theory, the fringes are produced by the interference between diffraction orders generated by the Ronchi ruling. Therefore, this test can be seen as a lateral shear interferometer, where the optical path difference (OPD) that generates the interference pattern is associated with the original wavefront slopes instead of a wavefront directly, as in others interferometers.[1,2]

1.1.1 *On-axis Ronchigrams*

Considering the typical geometry in the Ronchi test for analysis of convergent wavefronts, the transmittance for a ruling can be written as[2]

$$M(x_r) = 1 + \cos\left(\frac{2\pi x_r}{d} - \beta\right), \tag{1}$$

where d is the period of the ruling and β is a lateral shear parameter determined by the initial ruling position. The Ronchi ruling is usually fabricated with binary transmittance, nonsinusoidal, so that the binary transmittance $M(x_r)$ is generated by assigning a value of zero if $M(x_r) \leq 1$ or assigning a value of one if $M(x_r) > 1$.

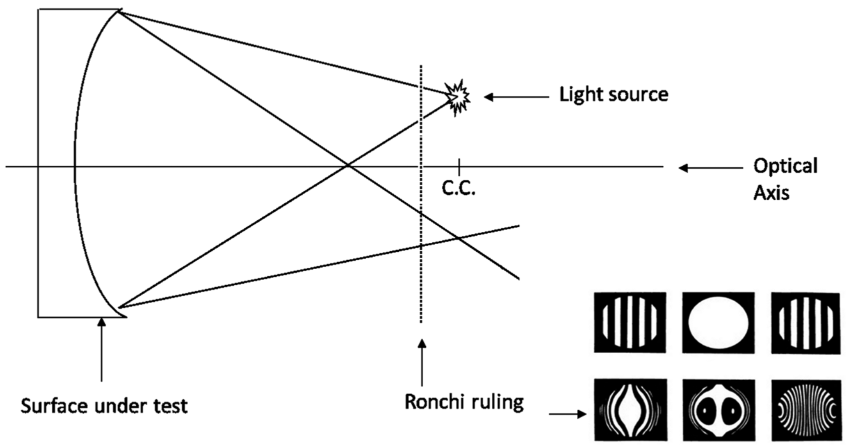


Fig. 1 Ronchi test basic configuration.

The irradiance $I(x,y)$ at any point of the Ronchigram can be understood as the irradiance associated to an interferogram and can be expressed as the OPD between the two sheared wavefronts. If a light detector is positioned on the observation plane, a distorted irradiance pattern will result due to the wavefront aberrations,[2] given by

$$I(x,y) = a(x,y) + b(x,y)\cos\left(\frac{2\pi \mathrm{OPD}}{\lambda}\right) + c(x,y), \tag{2}$$

where $a(x,y)$ and $b(x,y)$ are the background illumination and local contrast, respectively, the function $c(x,y)$ corresponds to the noise introduced into the interferogram, λ is the wavelength of light used, while the OPD in the lateral shear interferometer corresponds to the difference between the original wavefront and the sheared one[1,6,7] and is equal to

$$\mathrm{OPD} = W(x,y) - W(x+\Delta x, y), \tag{3}$$

where Δx is the lateral shear of the first diffraction order in the x direction, $W(x,y)$ represents the original wavefront which can be expressed by a polynomial function (i.e., Zernike, Seidel, Kingslake, etc.), or by the sagitta difference between the conical surface under test ($z(x,y)$ and the osculating sphere ($z_o(x,y)$),[1] then

$$2W(x,y) = z(x,y) - z_o(x,y). \tag{4}$$

The sagitta can be described by

$$z = \frac{c\rho^2}{1+\sqrt{1-(k+1)c^2\rho^2}}, \tag{5}$$

where c is the inverse of the radius of curvature, ρ^2 are the spatial coordinates of the pupil (x^2+y^2), and k is the conic constant of the surface according to Table 1.

Substituting Eq. (3) into Eq. (2) we obtain an equation that describes the two-dimensional distribution of the irradiance for a lateral shear interferometer, which is the same equation for the irradiance produced in the classical Ronchi test,

$$I(x,y) = a(x,y) + b(x,y)\cos\left[\frac{2\pi(W(x,y) - W(x+\Delta x, y))}{\lambda}\right] + c(x,y) \tag{6}$$

with the factor Δx equal to

$$\Delta x = \frac{\lambda r}{d}, \tag{7}$$

where r is the paraxial radius of curvature of the surface under test, d is the period of the Ronchi ruling, and λ is the wavelength used, in our case $\lambda = 0.55\ \mu$m.

1.1.2 *Off-axis Ronchigrams*

The off-axis surfaces play an important role in optics, especially in the design of instruments, such as spectrometers, light concentrating systems, etc., allowing unrestricted access to the focal point at certain deviation angle.[8,9] These surfaces have been used in general to focus an incident beam of light outside the beam's path, eliminating transmission losses and diffraction effects.

Following the procedure described by Cardona et al.[8] and Izazaga et al.[9] we can find the equations for the sagitta in power series, for an either on-axis or off-axis conic section. These expressions are derived by applying transformations to the coordinate systems, by rotation and translation, in order to align this new coordinate system with the normal at the central point of the off-axis section (see Fig. 2).

Table 1 Values of conic constants for conic surfaces.

Type of conic surfaces	Conic constant value (k)
Hyperboloid	$k < -1$
Paraboloid	$K = -1$
Ellipsoid rotated about its major axis	$-1 < k < 0$
Sphere	$K = 0$
Ellipsoid rotated about its minor axis	$k > 0$

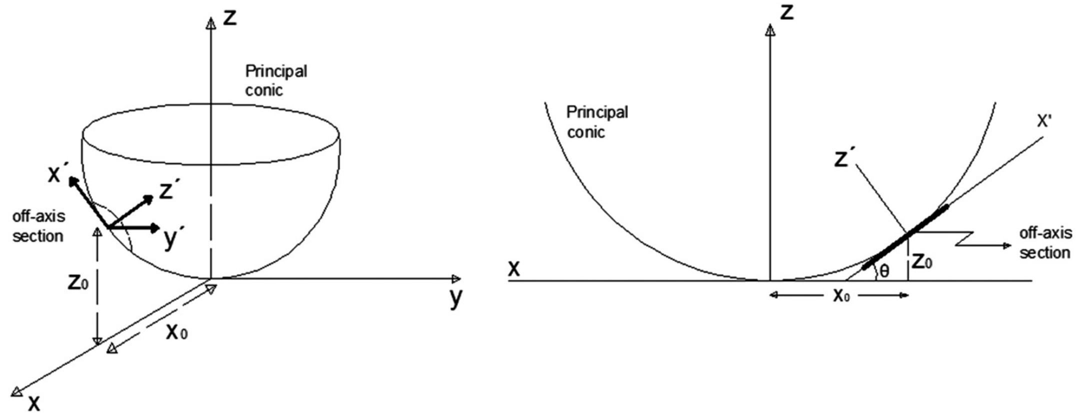


Fig. 2 Off-axis conic section described by its basic parameters.

Finally, applying the necessary transformations and expanding the resulting terms in power series, the equation for the sagitta is

$$z \approx \frac{1}{2}c\delta(\delta^2x^2+y^2)+\frac{1}{2}c^2\delta^2kx\sin\theta\cos\theta(\delta^2x^2+y^2)$$
$$+\frac{1}{2}c^3\delta^3(\delta^2x^2+y^2)(\delta^2b(1+3c\delta kx\sin\theta\cos\theta)x^2+b(3c\delta kx\sin\theta\cos\theta)y^2), \tag{8}$$

where $b=1+k\cos^2\theta$ and $\delta=\sqrt{1+k\sin^2\theta}$. A similar procedure can be done for on-axis conic surface [see Eq. (5)], and obtain an equation with the same degree of approximation, as

$$Z_A \approx \frac{1}{2}c'(x^2+y^2)+\frac{1}{8}(k'+1)c'(x^2+y^2)^2. \tag{9}$$

The sagitta difference between on-axis and off-axis sections is $2W(x,y)=Z_A-z$.

Transforming the coordinates system to a cylindrical one ($x=\rho\cos\phi$, $y=\rho\sin\phi$), and developing the full polynomial for the wavefront $W(x,y)$, [8,9] then

$$W(x,y)=\alpha_{20}\rho^2+\alpha_{22}\rho^2\cos 2\phi+\alpha_{31}\rho^3\cos\phi+\alpha_{33}\rho^3\cos 3\phi+\alpha_{40}\rho^4+\alpha_{42}\rho^4\cos 2\phi+\alpha_{44}\rho^4\cos 4\phi+\ldots, \tag{10}$$

where the aberration coefficients are defined by

$$\alpha_{20}=\frac{c}{2}\left[\frac{c'}{c}-\frac{\delta}{2}(1+\delta^2)\right] \quad \text{focus;}$$
$$\alpha_{22}=1/4c\delta(1-\delta^2) \quad \text{astigmatism;}$$
$$\alpha_{31}=-1/4c^2\delta^2k\sin\theta\cos\theta(1+\delta^2) \quad \text{coma;}$$
$$\alpha_{33}=1/4c^2\delta^2k\sin\theta\cos\theta(1-\delta^2) \quad \text{trefoil;}$$
$$\alpha_{40}=\frac{c^3}{8}\left[\frac{(k'+1)c'^3}{c^3}-\frac{1}{4}\delta^3b(1+\delta^2)^2-\frac{1}{8}\delta^3b(\delta^2-1)^2\right] \quad \text{spherical;}$$
$$\alpha_{42}=-1/16c^3\delta^3b(\delta^4-1)] \quad \text{sec .astigmatism;}$$
$$\alpha_{44}=-1/64c^3\delta^3b(\delta^2-1)^2 \quad \text{tetrafoil.} \tag{11}$$

2 Ronchigram Analysis with ReRRCA Algorithm

For the Ronchigram analysis of both on-axis and off-axis surfaces, the Ronchigrams recovery with random aberrations coefficients (ReRRCA)[7] follow the process shown in the flowchart of Fig. 3.

2.1 *Step 1: Reading the Ronchigram and Fringe Skeletonizing*

The first step is skeletonizing the Ronchigram to be analyzed in order to work only with the most important features of the image, i.e., working with Ronchigram's fringe centroids. It is noteworthy that the skeletonizing process is the only method for image enhancement that is used for the present analysis.

The procedure for fringe skeletonizing is scanning each row of the fringes image, by finding the maxima corresponding to each fringe; this recognition is to find the positive and negative slopes of each irradiance profile.

2.2 *Step 2: $W(x,y)$ Proposed and Ronchigram Generation*

The recovery of the aberration coefficients for the third-order polynomial of the synthetic wavefront is achieved by assigning random values to the aberration coefficients associated with the OPD of the recovered wavefronts. The algorithm has two main cycles that are described below.

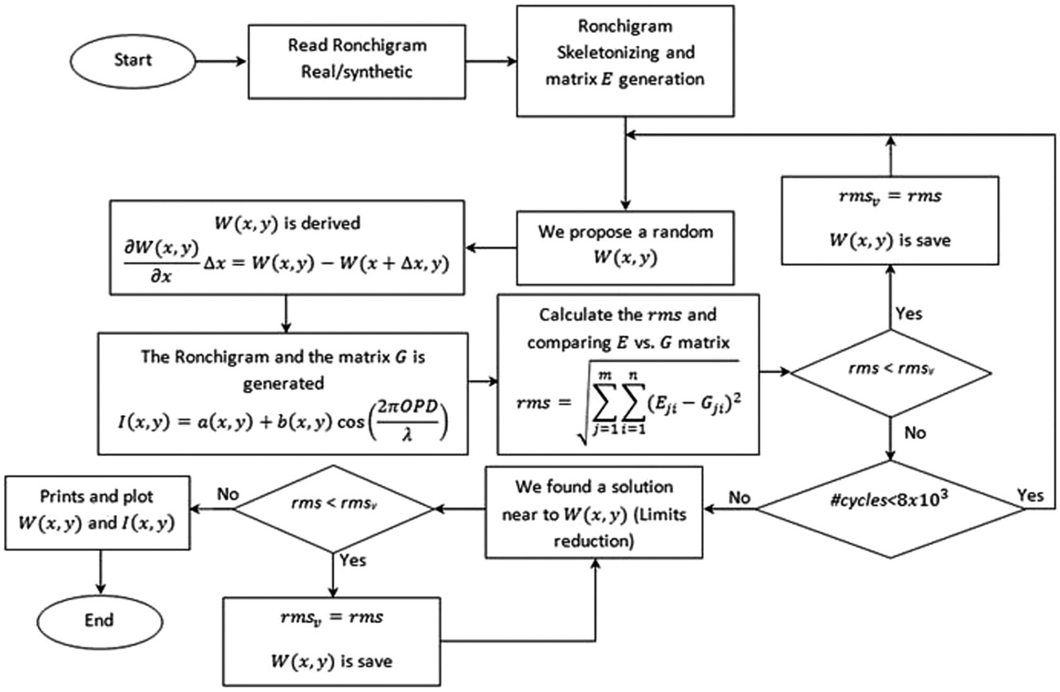


Fig. 3 Flow chart of the proposed algorithm.

2.2.1 *Approximation cycle*

This cycle is responsible for finding the closest aberration coefficients to the solution for synthetic wavefront. This is done by generating as many Ronchigrams as desired, with random coefficients. In our analysis were generated 8000; these Ronchigrams are generated between an established range, in our case -2λ to 2λ ($\lambda = 5.5 \times 10^{-5}$ cm), since the generated functions are smooth. From the comparison between the synthetic and recovered Ronchigram's coefficients, an root mean square (rms) value is obtained and a new analysis cycle can be started.

After having the fringe skeleton of the Ronchigram tested, each maximum position is found line by line, thus generating a matrix called $E_{\text{synthetic}}$, where the number of rows corresponds to the lines in the analyzed Ronchigram, while the number of columns corresponds to the number of maximum encountered by analyzed line, and a matrix is found as

$$E_{\text{synthetic}} = \begin{pmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{m1} & E_{m2} & \cdots & E_{mn} \end{pmatrix}. \tag{12}$$

Every time you generate a new simulated Ronchigram, the positions of the maxima are found, thus generating another matrix called $G_{\text{recovered}}$.

$$G_{\text{recovered}} = \begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{pmatrix}. \tag{13}$$

With both matrix $E_{\text{synthetic}}$ and $G_{\text{recovered}}$, the rms difference is calculated by means of

$$\text{rms} = \sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}(E_{ji} - G_{ji})^2}, \tag{14}$$

where n is the number of fringes measured and m is the number of lines used.

The Ronchigram generated that has the least rms will be taken as recovered Ronchigram by ReRRCA algorithm or the resulting Ronchigram from the analysis.

2.2.2 *Better adjustment for the rms value (limits reduction cycle)*

This cycle is responsible for optimizing the coefficients difference rms found by the approximation cycle. This step is done in the following way; select one of the coefficients and change it while maintaining the other coefficients fixed, as shown in Fig. 4.

A variable Q is defined that takes values equal to 1×10^n, where $n = 1, 2, 3, \ldots, p$; and p is the number of significant digits of each coefficient. Whenever Q takes a value (either positive or negative) the rms is calculated; if this is lower than the previous rms, the coefficient takes the new value and the cycle begins again; in the case that the rms is greater, the cycle is repeated with the old value of the coefficient.

3 Validation for ReRRCA

For the ReRRCA algorithm validation, different types of Ronchigrams were simulated, including on-axis and off-axis optical surfaces. This is done in order to check that the retrieved aberration polynomial coefficients, generated with the simulated Ronchigrams, correspond to the coefficients of the analyzed Ronchigram. The Ronchigrams generated have a size of 501×501 pixels.

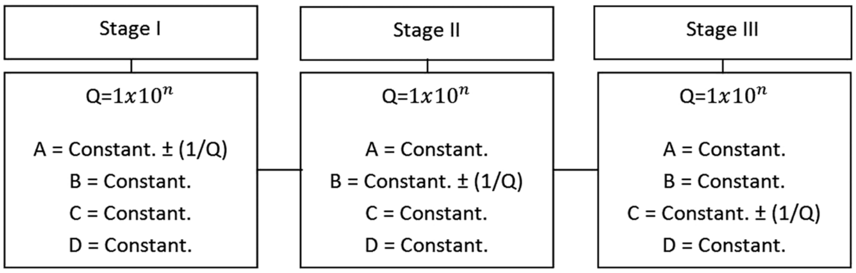


Fig. 4 Diagram showing the operation of the limits reduction cycle.

3.1 *Synthetic Ronchigrams of a Spherical Surface*

The first step in the validation of the algorithm was generating a synthetic Ronchigrams of a spherical surface, which only present Gaussian noise and defocus effect [Fig. 5(a) and 5(b)]. This is because it is well known that for a perfect spherical surface with a defined focus, only straight fringes appear in the irradiance pattern, with parallel and equally spaced fringes, and with the same number of fringes in both x and y direction, as shown in Fig. 5(a) and 5(b).

The Ronchigrams recovered by the proposed algorithm are shown in Fig. 5(c). and 5(d). The aberration polynomial coefficients retrieved for these same images are described in Table 2.

It is noteworthy that the focus coefficients remain constant for the simulated and recovered Ronchigrams. Given that this value of focus may remain constant since it only depends on the position of the grating l' and the paraxial radius of curvature paraxial r of the surface under test as

$$\text{Focus} = \frac{l'}{r^2}. \tag{15}$$

For all retrieved Ronchigrams in this paper an rms value less than 10 was obtained.

3.2 *Synthetic Ronchigrams of an On-Axis Hyperbolic Surface*

In this subsection, a hyperbolic surface with radius of curvature of 53.3 cm, 7.32 cm diameter and a conic constant equal to −3.65, the grid classical Ronchi that was used was of

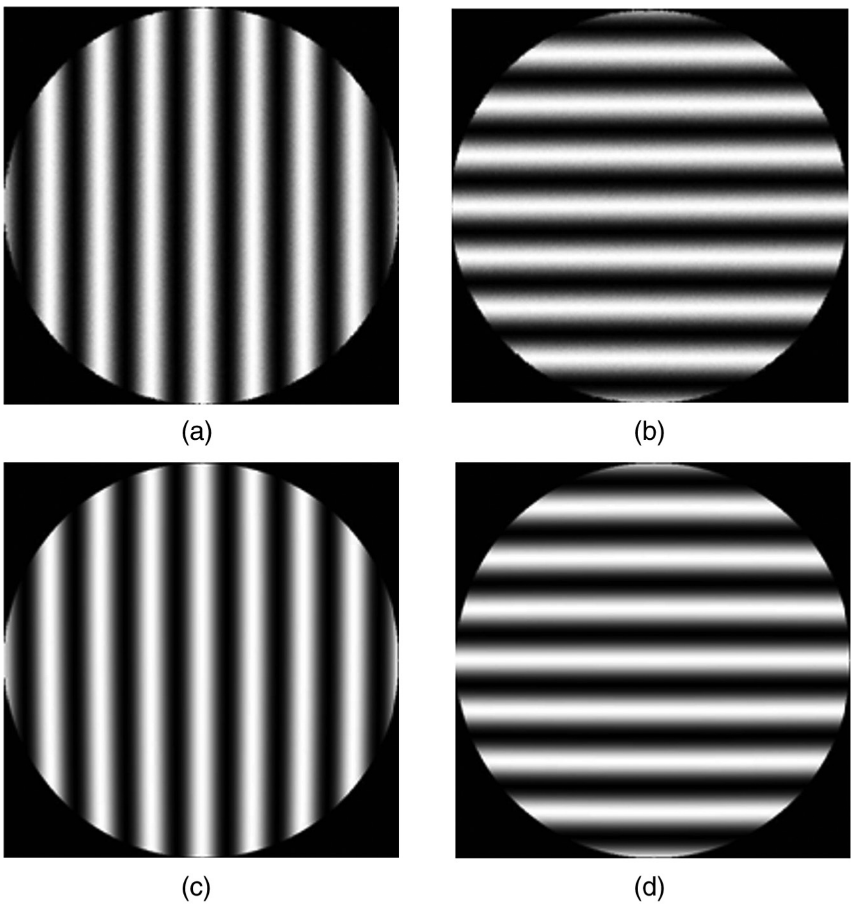


Fig. 5 Images (a) and (b) are noise simulated Ronchigrams, introducing only focus, the fringes are parallels to the x and y axes, respectively. Images (c) and (d) are Ronchigrams recovered by the proposed algorithm.

Table 2 Comparison between the synthetic Ronchigram coefficients and the recovered by the algorithm.

Aberration	Synthetic x, y	Recovered x	Recovered y	Difference at x	Difference at y
Spherical	0.0λ	0.002λ	0.003λ	0.002λ	0.003λ
Coma	0.0λ	0.003λ	0.004λ	0.003λ	0.004λ
Astigmatism	0.0λ	0.002λ	0.003λ	0.002λ	0.003λ
Focus	17.128λ	17.128λ	17.128λ	0.0λ	0.0λ

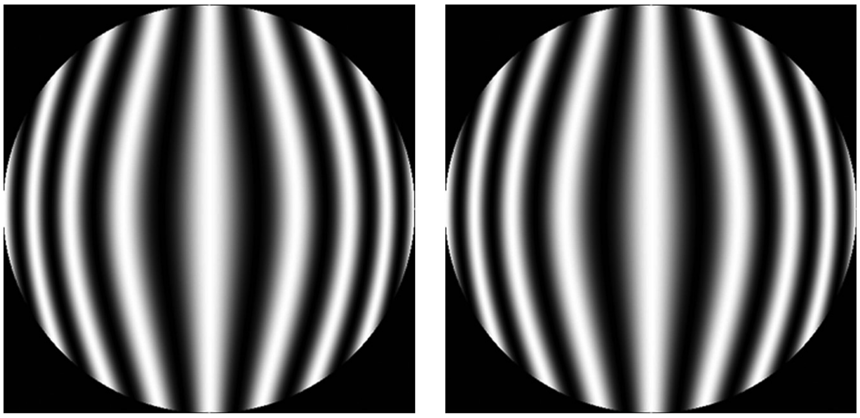

(a) (b)

Fig. 6 (a) Synthetic Ronchigram of an on-axis hyperbolic surface (inside of focus), (b) Ronchigram recovered by Ronchigrams recovery with random aberrations coefficients (ReRRCA).

80 lpi, at a distance of 53 cm from the vertex of the mirror. The aberrations of the on-axis hyperbolic wavefront are focus and spherical; this latest can be calculated for any surface[10] as

$$\text{Spherical} = \frac{|k|}{8r^3}. \tag{16}$$

To calculate the absolute impact in the surface error, we can calculate a surface rms by means of

$$\text{rms}_{\text{surf}} = \frac{1}{N}\sqrt{\sum [W(x,y)_{\text{simulated}} - W(x,y)_{\text{recovered}}]^2}. \tag{17}$$

The results obtained of the on-axis hyperbolic surface are shown in Fig. 6(a) and 6(b). The coefficients retrieved by the proposed algorithm are shown in Table 3. As shown, the recovered coefficients by ReRRCA are satisfactory because the maximum absolute error in the coefficients is $1.1\times 10^{-3}\lambda$, where the difference between the synthetic and the recovered surfaces gives an rms_{surf} of $4.4\times 10^{-3}\lambda$.

We simulated another hyperbolic surface with a radius of curvature of 60 cm, 15 cm diameter and a conic constant equal to -3.65, the classical Ronchi ruling used was of 50 lpi, at a distance of 62.1 cm from the vertex of the mirror. The results obtained are shown in Fig. 7(a) and 7(b). The coefficients retrieved are shown in Table 4. The maximum absolute error in the coefficients is $2.25\times 10^{-3}\lambda$ and an rms_{surf} of $8.52\times 10^{-4}\lambda$. Figure 8 shows the behavior of the proposed algorithm when the merit function is minimized during the optimization process; it also shows the behavior of the two main cycles of ReRRCA where the first cycle (of approximation) finds coefficients close to the solution coefficients. This is because it includes a random function, so the probability that in one trial all generated coefficients are equal to the analyzed coefficients is almost null, while the

Table 3 Comparison between the coefficients of an on-axis simulated Ronchigram (inside of focus) and the recovered by Ronchigrams recovery with random aberrations coefficients (ReRRCA).

Aberration	Synthetic (calculated by formula)	Recovered by ReRRCA	Absolute error
Spherical	$5.58\times 10^{-2}\lambda$	$5.44\times 10^{-2}\lambda$	$6.0\times 10^{-4}\lambda$
Coma	0.0λ	$1.1\times 10^{-3}\lambda$	$1.1\times 10^{-3}\lambda$
Astigmatism	0.0λ	$1.0\times 10^{-3}\lambda$	$1.0\times 10^{-3}\lambda$
Focus	1.504λ	1.504λ	0.0λ

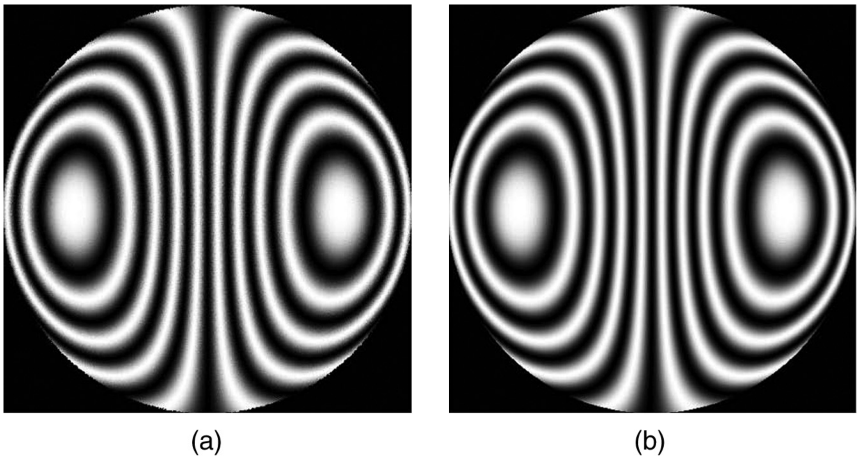


Fig. 7 (a) Synthetic Ronchigram of an on-axis hyperbolic surface (outside of focus), (b) Ronchigram recovered by ReRRCA.

Table 4 Comparison between the coefficients of an on-axis simulated Ronchigram (outside of focus) and the recovered by ReRRCA.

Aberration	Synthetic (calculated by formula)	Recovered by ReRRCA	Absolute error
Spherical	$3.64 \times 10^{-2}\lambda$	$3.66 \times 10^{-2}\lambda$	$2.0 \times 10^{-4}\lambda$
Coma	0.0λ	$2.25 \times 10^{-3}\lambda$	$2.25 \times 10^{-3}\lambda$
Astigmatism	0.0λ	$1.96 \times 10^{-3}\lambda$	$1.96 \times 10^{-3}\lambda$
Focus	5.303λ	5.303λ	0.0λ

second cycle (limits-reduction), which modifies the coefficients in a controlled manner, finds the closest approximation to the final solution.

To obtain statistical results of the behavior of the proposed algorithm we analyze this inside of focus Ronchigram in several trials. Table 5 shows the mean and standard deviation (σ) of the aberration coefficients after 11 trials.

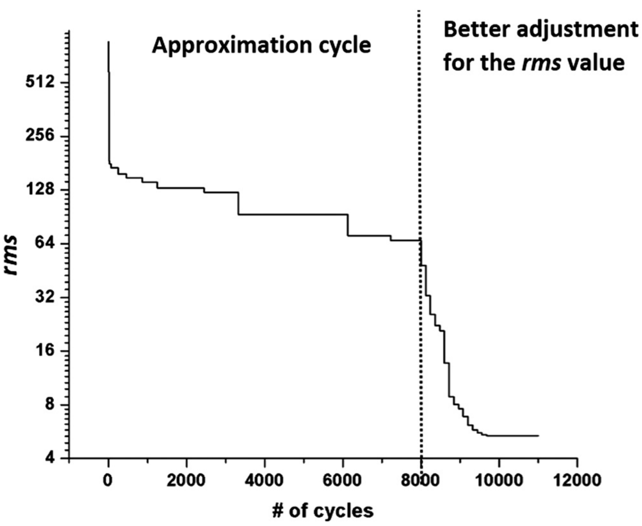


Fig. 8 Behavior of the rms value for the on-axis hyperbolic surface (outside of focus).

3.3 *Synthetic Ronchigrams of an Off-Axis Hyperbolic Surface*

For the simulation of the Ronchigrams of an off-axis hyperbolic surface, the Ronchi classical ruling was placed at 29.5 cm from the vertex of the mirror, while the surface was simulated with 4 cm of diameter, 30 cm paraxial radius of curvature, 10 cm off-axis and a conic constant equal to −4.5. Figure 9(a) and 9(b) is the analyzed-synthetic and the recovered by ReRRCA Ronchigrams, respectively. The coefficients retrieved by the proposed algorithm are shown in Table 6, where it can be seen that the higher coefficients difference corresponds to astigmatism, and $\mathrm{rms}_{\mathrm{surf}}$ with 0.1245λ.

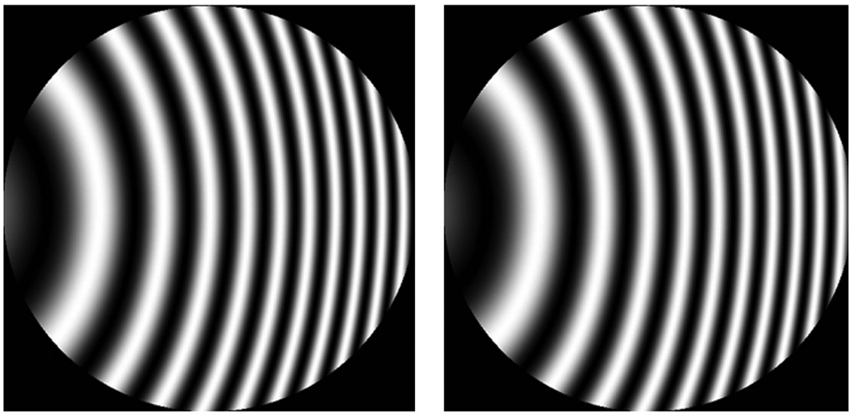
The coefficients of the synthetic Ronchigrams were calculated using Eq. (11), where all these coefficients depend on the characteristics of the mirror and the measured value from the optical axis of the parent surface to the center of the off-axis surface.

We simulated a Ronchigram with closed fringes of a hyperbolic surface with an off-axis distance of 6 cm (see Fig. 10), with radius of curvature of 60 cm, 10 cm diameter and a conic constant equal to −3.5, the classical Ronchi ruling used was of 50 lpi, at a distance of 64 cm from the vertex of the mirror.

Table 7 shows the results from the analysis of a Ronchigram with closed fringes where the maximum percentage error is 10.22 giving an $\mathrm{rms}_{\mathrm{surf}}$ of $9.6 \times 10^{-3}\lambda$. Figure 11 shows the behavior of the proposed algorithm

Table 5 Mean and standard deviation of the aberration coefficients for several trials of the proposed algorithm for the on-axis case.

Coefficient	Mean (λ)	Standard deviation (σ)
Spherical	3.664×10^{-2}	2.1×10^{-4}
Coma	-1.99×10^{-3}	2.48×10^{-3}
Astigmatism	9.10×10^{-3}	1.14×10^{-2}
rms	4.46471	0.25746



(a) (b)

Fig. 9 (a) Synthetic Ronchigram of an off-axis hyperbolic surface (inside of focus), (b) Ronchigram recovered by ReRRCA.

Table 6 Comparison between the coefficients of an off-axis simulated Ronchigram (inside of focus) and the recovered by ReRRCA.

Aberration	Synthetic (calculated by the formula) (λ)	Recovered by ReRRCA (λ)	Percentage error (%)
Focus	−9.2163	−8.5253	7.49
Astigmatism	41.2372	39.8894	3.51
Coma	6.6134	6.2625	5.31
Trefoil	−1.3227	−1.2218	7.63
Spherical	0.1869	0.1770	5.29
Sec. Astig.	−0.0403	−0.0380	5.71
Tetrafoil	0.0020	0.0019	5.00

when the merit function is minimized during the optimization process.

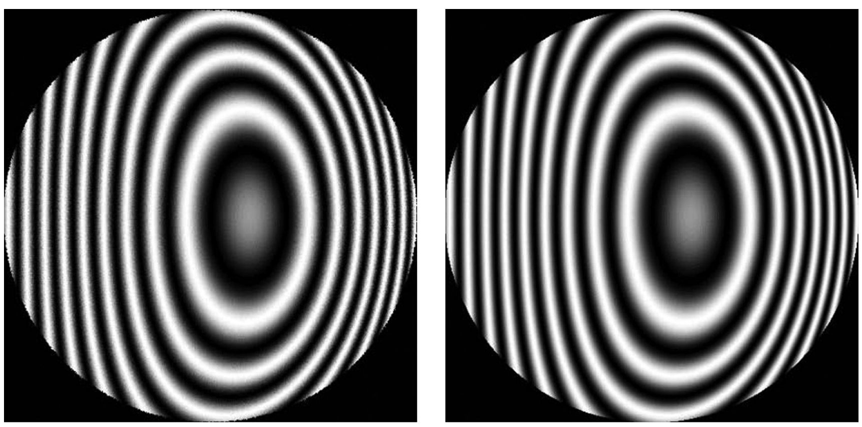
Table 8 shows the mean and standard deviation of the aberration coefficients for this case after 11 trials.

4 Error Produced by the Ruling Position

This section will discuss the error introduced in retrieving the aberration coefficients for the analyzed Ronchigrams with an error in the ruling position.

An error in the focus position reduces or increases the number of fringes present in the Ronchigram. Therefore, this error introduces an increase or decrease in the coefficient of astigmatism, given because the coefficient of astigmatism causes a similar effect on the Ronchigrams; in other words, when there is an error in the focus's coefficient, the astigmatism coefficient is modified in order to compensate this error.

Analyzing a Ronchigram generated by the interference produced by the diffraction orders diffracted along the x axis, the behavior of the error introduced at astigmatism is in the form



(a) (b)

Fig. 10 (a) Synthetic Ronchigram of an off-axis hyperbolic surface (outside of focus), (b) Ronchigram recovered by ReRRCA.

Table 7 Comparison between the coefficients of an off-axis simulated Ronchigram (outside of focus) and the recovered by ReRRCA.

Aberration	Synthetic (calculated by the formula) (λ)	Recovered by ReRRCA (λ)	Percentage error (%)
Focus	−4.999	−4.824	3.50
Astigmatism	2.518	2.465	2.11
Coma	0.821	0.811	1.24
Trefoil	-1.41×10^{-2}	-1.32×10^{-2}	6.38
Spherical	3.43×10^{-2}	3.29×10^{-2}	4.08
Astig. Sec.	-8.192×10^{-4}	-9.030×10^{-4}	10.22
Tetrafoil	3.522×10^{-6}	3.860×10^{-6}	9.59

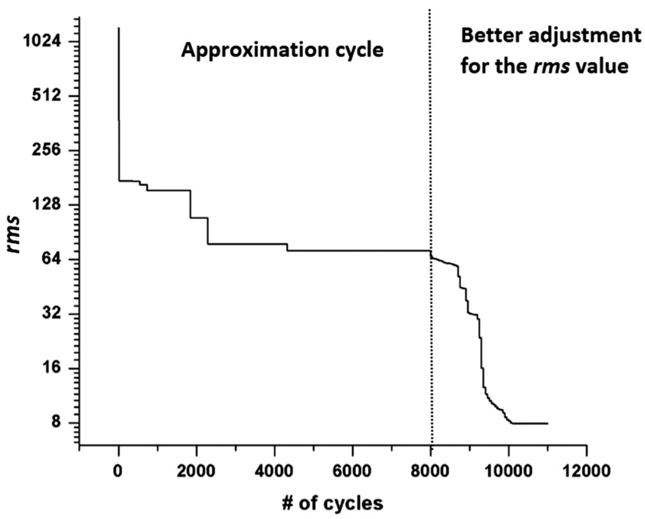


Fig. 11 Behavior of the rms value for the off-axis hyperbolic surface (outside of focus).

Table 8 Mean and standard deviation of the aberration coefficients for several trials of the proposed algorithm for the off-axis case.

Coefficient	Mean (λ)	Standard deviation (σ)
Astigmatism	2.10412	0.205601
Coma	0.82375	0.013870
Trefoil	-1.7773×10^{-2}	4.48×10^{-3}
Spherical	3.2808×10^{-2}	8.491×10^{-3}
Sec. Astig.	-6.9932×10^{-4}	1.86×10^{-4}
Tetrafoil	3.31182×10^{-6}	2.613×10^{-6}
rms	14.2849407	6.12972

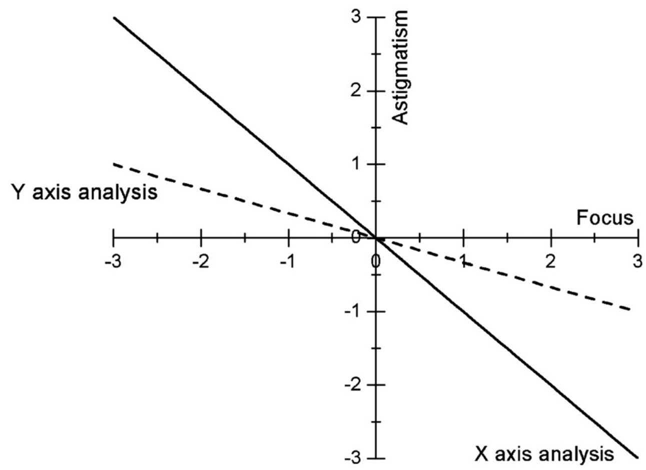


Fig. 12 Error introduced in the astigmatism coefficient depending upon an increase in the focusing effect.

$$\Delta\alpha_{22} = -\Delta\alpha_{20}. \tag{18}$$

While analyzing the Ronchigram with the diffraction orders along the y axis the error introduced is described by

$$\Delta\alpha_{22} = -\frac{\Delta\alpha_{20}}{3}. \tag{19}$$

These relationships can be seen in Fig. 12, where the solid line represents the analysis in the x direction, and the dashed line is for the y direction.

4.1 *Analysis Introducing a* $\Delta\alpha_{22}$

To test the focusing effect, two synthetic Ronchigrams are used without noise just introducing astigmatism aberration and focusing effect. The analysis of both Ronchigrams introduced a focusing error by $-\lambda$; the results of this analysis are presented below.

As seen in Fig. 13, the Ronchigrams recovered by the algorithm are exact (this is with rms = 0), but the coefficient of astigmatism are greater (Table 9). The absolute error of 1λ in the focus's coefficient of the Ronchigrams correspond to an $\mathrm{rms}_{\mathrm{surf}}$ on the surface of $9.04 \times 10^{-2}\lambda$, for Fig. 13(a), Fig. 13(b) gives an $\mathrm{rms}_{\mathrm{surf}}$ of $3.04 \times 10^{-2}\lambda$.

Note that Eqs. (17) and (18) are kept identical for analyzing Ronchigrams with different coefficients for spherical and coma.

5 Conclusions

This paper presents an algorithm for obtaining the aberration coefficients of a synthetic-proposed wavefront from a recovered wavefront Ronchigram. It has been shown that for the recovery of the synthetic wavefront only one Ronchigram is required in the analysis, given that a Ronchigram can only be generated by a fixed wavefront. This means that for a Ronchigram with a fixed focus, there is only one wavefront that satisfies the synthetic Ronchigram.

In the case of the analysis for on-axis conical surfaces the maximum absolute error observed in retrieving the aberration coefficients was $4 \times 10^{-3}\lambda$; while for off-axis surfaces the maximum absolute error observed was less than 0.1λ.

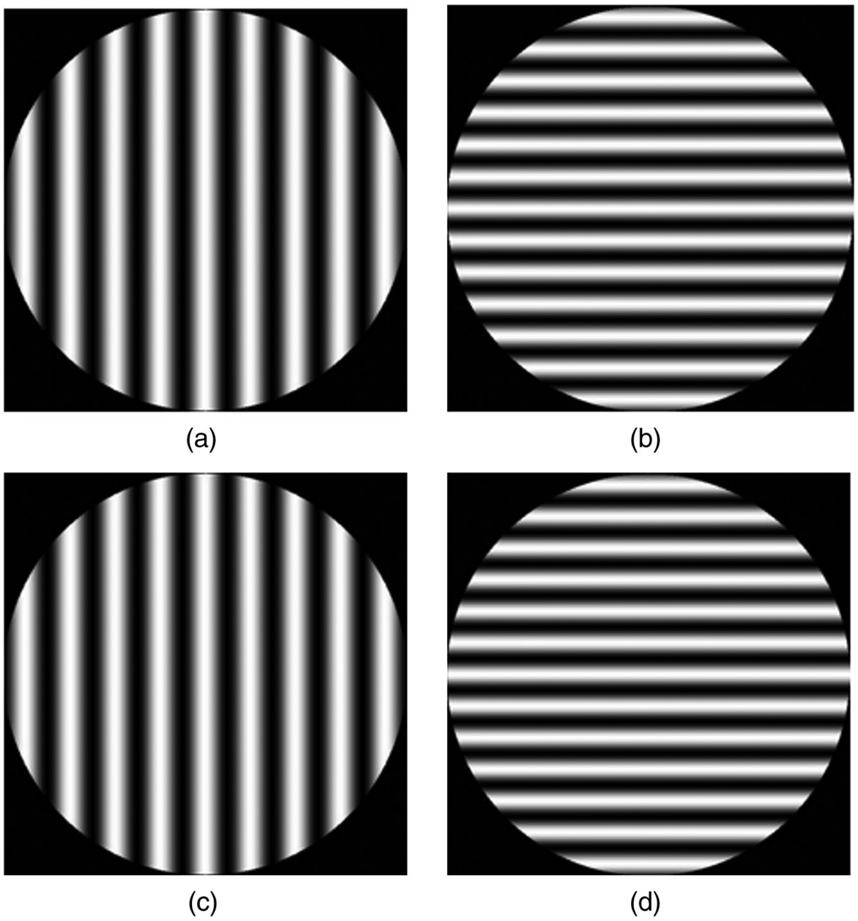


Fig. 13 The images (a) and (b) are noiseless synthetic Ronchigrams of a spherical surface, while (c) and (d) are the retrieved Ronchigrams by ReRRCA introducing a $\Delta\alpha_{22}$.

Table 9 Comparison between synthetic Ronchigram coefficients and recovered by the algorithm introducing a $\Delta\alpha_{22}$.

Aberration	Simulated in x, y	Recovered in x	Recovered in y	Error in x	Error in y
Spherical	0.0λ	0.0λ	0.0λ	0.0λ	0.0λ
Coma	0.0λ	0.0λ	0.0λ	0.0λ	0.0λ
Astigmatism	5.0λ	6.0λ	5.33λ	1.0λ	0.33λ
Focus	18.5λ	17.5λ	17.5λ	−1.0λ	−1.0λ

Finally, the analysis of the introduced error in recovering the aberration coefficients for the synthetic wavefront was analyzed as due to the Ronchi ruling position.

Acknowledgments

Daniel Aguirre-Aguirre, Rafael Izazaga-Perez, and Brenda Villalobos-Mendoza thank the CONACyT (México) graduate studies scholarships granted, CVU: 271299, 270587, and 272864, respectively. Fermín Granados-Agustín and Maria Percino-Zacarías thank the project promep UTMIX-CA-22 for the partial support.

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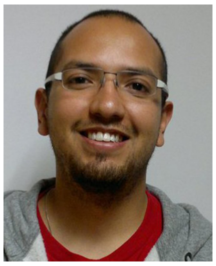

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