Waveguide properties of the asymmetric collision between two bright spatial solitons in Kerr media

D. Ramírez Martínez,[1] M. M. Méndez Otero,[1] M. L. Arroyo Carrasco,[1] and M. D. Iturbe Castillo[2,*]

[1]*Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Av. San Claudio y 18 Sur. Col San Manuel, C.P. 72570, Puebla, Puebla, México*

[2]*Instituto Nacional de Astrofísica, Óptica y Electrónica, Luis Enrique Erro # 1, C.P. 72840 Tonantzintla, Puebla, México*

[*]*diturbe@inaoep.mx*

Abstract: In this work, we numerically characterize the waveguide properties of the asymmetric collision between two bright spatial solitons in a nonlinear Kerr media. The results demonstrate that the energy carried by a probe beam guided by one soliton can be transferred after the collision to the waveguide created by the other soliton depending on the initial separation between the solitons, the angle of their collision, and in some cases, the particular soliton that initially guides the probe beam. The observed behavior is equivalent to that obtained for the symmetrical collision when there is an initial relative phase between the solitons.

OCIS codes: (190.0190) Nonlinear optics; (190.3270) Kerr effect; (190.6135) Spatial solitons.

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1. Introduction

Spatial solitons are optical beams that propagate without diffraction in a nonlinear media creating their own waveguide [1]. As a result it is possible to utilize the spatial solitons as optical channels for another beam in an intensity-dependent refractive-index medium [1–3]. The idea of using spatial solitons to guide probe beams was suggested theoretically by Chiao et al. [4], and proved experimentally by Luther-Davies and Yang [5]. These results had a direct effect on the development of logical and optical interconnect devices [6]. Previously, the soliton interaction between two coherent bright spatial solitons in Kerr media were studied and proved that it is possible to control a weak beam [7–9] and even obtain an optical switch, according to the phase difference between the solitons [10, 11].

It is well know that all collisions between solitons are fully elastic in Kerr media, which implies that the number of solitons is always conserved. Experiments demonstrating soliton interactions in (1 + 1)-D glass waveguides were performed in 1991 by Aitchison *et al.* [12,13]. They reported that in-phase Kerr solitons attracted one another, whereas out-of-phase solitons repelled one another. The situation is more complex for other relative phases. Shalaby *et al.* [14], showed that for a $\pi/2$ phase difference, one soliton gained energy at the expense of the other soliton. Furthermore, the energy exchange direction is switched when the phase is increased to $3\pi/2$. These effects can be viewed as the consequence of the well-known four-wave mixing term in nonlinear optics. These experiments demonstrated the basic properties of the coherent Kerr soliton interactions. In [15] Kerr spatial solitons are analyzed analytically and numerically for any collision angle demonstrating that they are robust.

It is important to understand the way in which the energy of a probe beam is transferred from one waveguide to another photoinduced by the solitons [16]. In previous papers, it was shown using numerical simulations that for certain parameters, such as the collision angle or the relative phases between the solitons, the waveguides induced by colliding solitons in a Kerr medium can act as a dynamic beam splitter for a weak probe beam, with switching ranging from total transmission to total deflection [6,16–18]. Recently the asymmetric collisions between bright and dark temporal solitons were studied and the results indicted several applications of the collisions to propose a photon trap, beam splitter, phase shifter and mode-separating beam splitter [16–18].

Although there are many papers considering collisions between solitons in media with nonlinearities more complicated than the Kerr one, very few of them study their waveguide properties such as energy and confined modes. An adequate knowledge of the basic guiding properties of soliton collisions in Kerr media allows one to identify which phenomena are due to more complicated nonlinear responses. In this work we report a numerical study of the waveguide properties of the asymmetric collision between two spatial solitons in Kerr media as a function of the separation distance and the solitons collision angle.

2. Theoretical model

We followed the same approach presented in Ref [6], where the simultaneous propagation of an intense, soliton, (q_1) and a weak, probe, (q_2) beam in a Kerr media were described by the following coupled equations:

$$i\,\frac{\partial q_1}{\partial Z} = \frac{1}{4}\frac{\partial^2 q_1}{\partial X^2} \pm \frac{L_D}{L_{NL}}\left|q_1\right|^2 q_1, \qquad (1)$$

$$i\,\frac{\partial q_2}{\partial Z} = \frac{1}{4} r_n r_k \frac{\partial^2 q_2}{\partial X^2} \pm \frac{2L_D}{r_k L_{NL}}\left|q_1\right|^2 q_2, \qquad (2)$$

where q_j is the amplitude of the optical field normalized to the maximum intensity I_{mj}, $L_D = n_{01}k_{01}x_{01}^2/2$ is the diffraction length with n_{01} the linear refractive index, k_{01} the wave number and x_{01} the initial beam width for beam q_1, $L_{NL} = (|n_2|k_0I_m)^{-1}$ is the nonlinear length with n_2 is the nonlinear refractive index, $r_n = n_{01}/n_{02}$ with n_{02} the linear refractive index for beam q_2, and $r_k = k_{01}/k_{02}$ with k_{02} the wave number for beam q_2. In this work, by simplicity we used $r_n = r_k = 1$, this means that the soliton and probe beam have the same wavelength and direction (experimentally, different polarizations can be used to differentiate both beams), the results considering other values of these parameters give similar behavior.

Equation (1) is the nonlinear Schrödinger equation, which it has been analytical studied by the inverse scattering method and is well known that admits two stationary soliton type solutions:

$$q_1(X,Z) = sech(\sqrt{2}X)exp(-iZ\ /\ 2), \tag{3}$$

and

$$q_1(X,Z) = \tan h(\sqrt{2}X)exp(-iZ). \tag{4}$$

These solutions are known as the bright (Eq. (3)) and dark (Eq. (4)) solitons, for a positive and a negative nonlinear refractive index material, respectively.

To investigate the waveguide properties of the collision of solitons we considered as initial condition two identical spatial solitons beams, where one soliton (S1) propagated along the Z axis and other soliton (S2) is initially separated by a distance c from first soliton and makes an angle θ with respect to the Z axis. The following equation was used as initial condition:

$$q_1(X) = sech(2^{1/2}X) + sech(2^{1/2}(X+c))exp(-iV(X+c)), \tag{5}$$

where $tan\theta = V/2$. Note that this initial condition is equivalent to the symmetric collision but with a relative phase difference between the solitons equal to $Vc/2$. As a probe beam we considered a Gaussian beam, with unitary amplitude, as initial condition for Eq. (2), given by

$$q_2(X) = exp(-wX^2), \tag{6}$$

where w is a factor to adjust the width of the beam.

The coupled Eqs. (1) and (2) were numerically solved using a split-step method implemented in Matlab on a PC computer. The study considered only solitons with equal amplitude and width.

3. Numerical results

Considering that initially we have two beams, as that given by Eq. (5), condition that we called asymmetric collision, propagating in a positive Ker nonlinear media we analyzed the influence of the initial separation and collision angle on the behavior of a probe beam sent along the trajectory of one of the solitons. As it is well known the solitons are not affected by the collision, they emerged with the same amplitude and width. There was a small shifting in their positions but the rest was kept without change, see Fig. 1.

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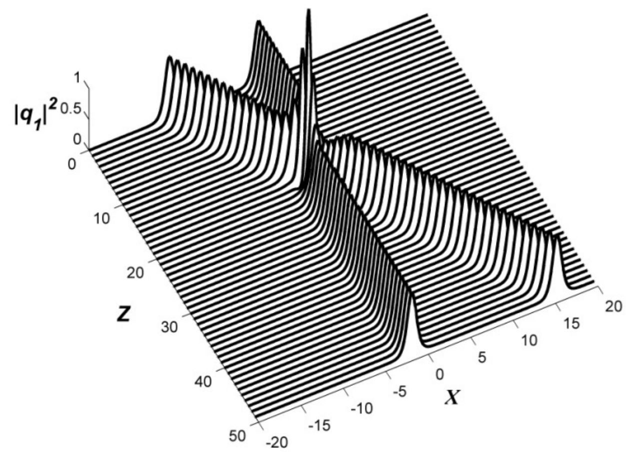

Fig. 1. Asymmetric collision of two bright spatial solitons with $V = 1$.

The trajectory and amount of energy at the end of the propagation of the probe beam depended on its initial position, initial distance between the solitons and collision angle. In Fig. 2 we show the evolution of the probe beam, when was set initially along the soliton that makes and angle θ with Z axis, for different separation distances c with $V = 1$. We can observe that the amount of light in each waveguide depended on the separation distance. For example in Figs. 2(a) and 2(d) for a distance of $c = 10.5$ and $c = 16$, respectively, the energy of the probe beam is almost entirely transferred to other waveguide; for a initial separation of $c = 11.5$, the amount of energy in each waveguide is almost the same (Fig. 2(b)); and for $c = 12.5$, Fig. 2(c), the probe beam energy was mainly retained in the initial waveguide.

The normalized amount of energy (E_i) in each waveguide at the end of propagation as a function of the initial separation c between the solitons when $V = 1$ is shown in Fig. 3. Separation distances $c<4.5$ where not considered because the overlap between the beams was large. The probe beam was co-centered with the soliton that propagates with an angle θ to the z-axis (we identify this soliton as S2). For values $c<9$ the energy was transferred to the waveguide photo-induced by the other soliton (S1). For separations between $10<c<15$ both waveguides shared different amounts of energy after the collision, except for $c = 12.5$ where almost all the energy was kept in the initial waveguide. For values $c>16$ the behavior described for the final amount of energy in each waveguide is repeated. For this collision angle the probe energy in each soliton waveguide is shown in Fig. 3. The results are the same as that obtained for symmetric soliton collisions when $|V| = 0.5$ (for both solitons) and the relative phase between them is varied from 0 to 2π radians. Our result is significant because demonstrates that in the asymmetric collision the amount of energy in each waveguide can be controlled with the initial separation between the solitons while keeping the collision angle constant without altering the relative phase between the solitons. We note that in previous works the energy after the asymmetric collision could also be controlled using an initial phase difference between the solitons [16].

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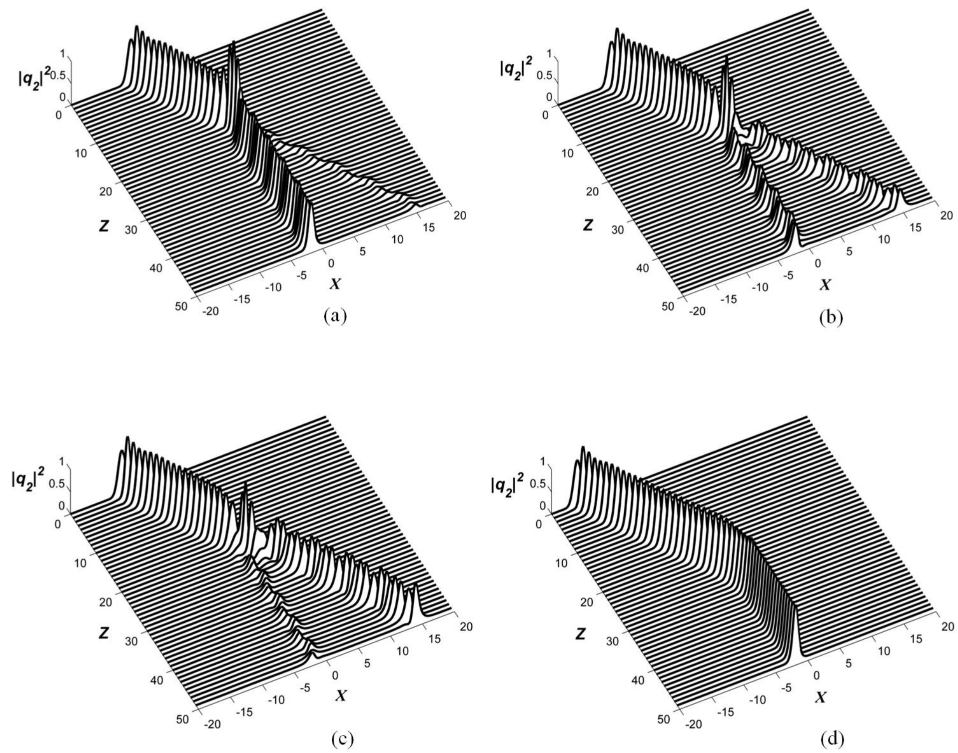

Fig. 2. Propagation of a probe beam in the waveguide photo-induced by soliton S2 with $V = 1$, and initial separations c: (a) 10.5, (b) 11.5, (c) 12.5, (d) 16.

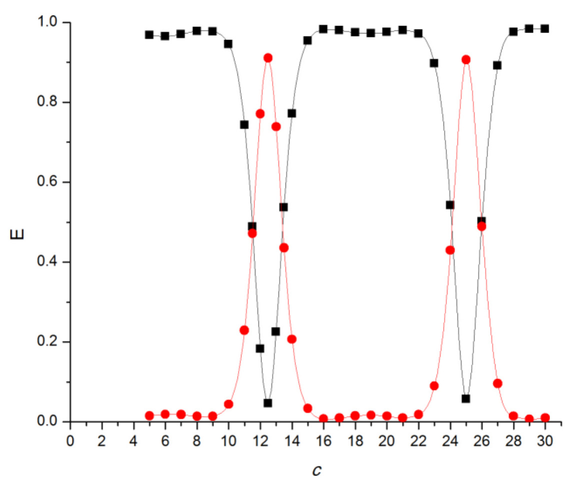

Fig. 3. Normalized confined energy E_1 (square) for the waveguide photo-induced by soliton S1 and E_2 (circle) for the waveguide photo-induced by soliton S2 as function of c, for an asymmetric collision with $V = 1$ when the probe beam was initially confined by S2.

Recently it was reported that the collision between temporal bright solitons can be used as a beam splitter and a mode-separating beam splitter [16–18]. In order to test the analogy with spatial solitons we show that it is possible to obtain a mode separator. We introduce as an

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initial condition, for the probe beam, a field distribution given by the sum of a sech mode with a high order mode [19] given by:

$$q_1(X) = [sech(2^{1/2}(X+c)) + (X+c)sech(2^{1/2}(X+c))]exp(-iV(X+c)). \qquad (7)$$

In Fig. 4(a) we show the behavior of such initial condition for the asymmetric collision of bright solitons with $V = 1$ and $c = 8$. We can observe that after the soliton collision the modes are separated, the fundamental mode is deflected while the higher mode is not. For $V = 1$ and initial separations between the beams of $10<c<15$ the splitter was not very efficient. This behavior was independent of which solitons guided the probe beam.

The asymmetric collision of solitons presented another interesting feature not mentioned before (for collision angles larger than 1): the amount of energy confined in each waveguide depends on which soliton initially guided the probe beam. In order to demonstrate numerically this we present, in Fig. 5, the behavior of the probe beam for an asymmetric collision when $V = 2$ and $c = 8$ between the solitons: in (a) when the probe beam was initially guided by S1 and in (b) when it was initially guided by S2. We observed that at the end of the propagation path in both cases the major amount of energy was confined in the other waveguide, however in Fig. 5(a) this energy is less than in Fig. 5(b). It is important to note that the light confined in the case of Fig. 5(a), after the collision, is not the fundamental mode, in support of the idea that the asymmetric collision can be used as a mode separator [17]. As for a Gaussian, when initial condition for the probe beam was given by Eq. (7), differences in the amount of energy and field distribution were observed when the probe beam was initially guided by one or the other soliton (see Fig. 6).

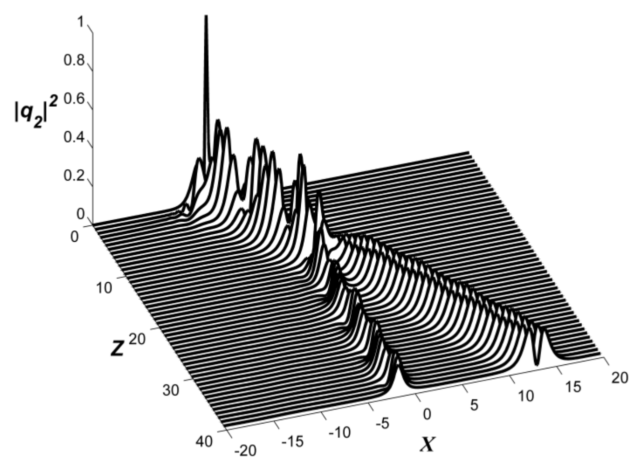

Fig. 4. Propagation of the probe beam given by the sum of a sech mode and a high order mode when was initially guided by S2 for the asymmetric collision of solitons with $V = 1$ and $c = 8$.

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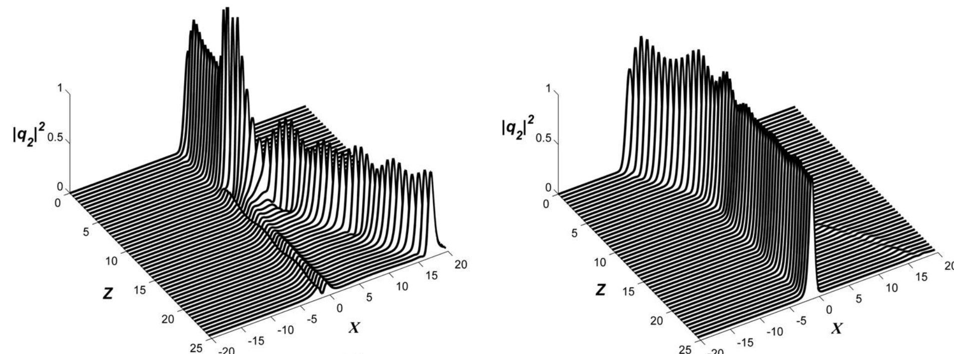
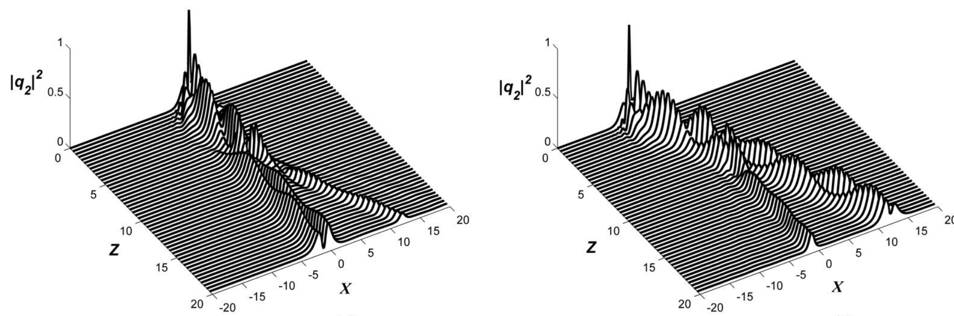


(a) (b)

Fig. 5. Propagation of the probe beam, with $V = 2$ and $c = 8$, when it was initially guided by: (a) S1 and (b) S2.



(a) (b)

Fig. 6. Propagation of a probe beam given by the sum of a sech mode and a high order mode for an asymmetric collision with $V = 2$ and $c = 8$ when the probe beam was initially guided by: (a) S1 and (b) S2.

A detail study of the dependence of the amount of energy confined in each waveguide for $V = 2$ as a function of the separation is shown in Fig. 7, where the probe beam was initially guided by: a) soliton S1 and b) soliton S2. The general behavior of both cases is similar; a difference was obtained in the maximum amount of energy confined in each waveguide. However, when the final confined energy, of the waveguide where the probe beam was initially guided, is plotted as a function of the separation c differences in magnitude and position are very clear, see Fig. 7(c). Larger values of V produced similar results with differences in: the separation between the solitons, the amplitude and mode of the confined field.

Adequate knowledge of the waveguide properties of the asymmetric collision of solitons enables a prediction of the behavior for more complicate configurations. For example we reproduce, in the spatial case, the phase shifter configuration proposed in Ref [17]. In Fig. 8 we present the collision of three solitons, two with transversal velocity zero and one with $V = 1$ and the behavior of a probe beam initially guided by the central soliton. In our case the initial relative phase between the solitons was zero and the separation was chosen to give the best energy transference ($c = 8$).

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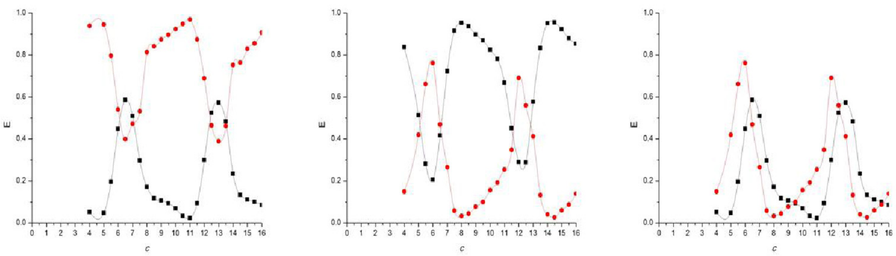
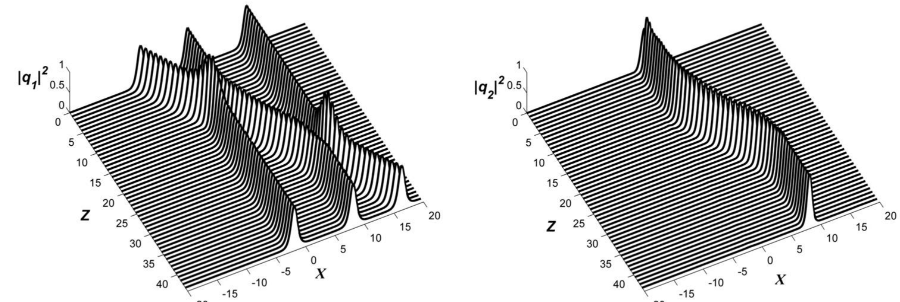


(a) (b) (c)

Fig. 7. Normalized confinement energy E_1 (square) for the waveguide photoinduced by soliton S1, and E_2 (circle) for the waveguide photo-induced by soliton S2 as function of *c*, for an asymmetric collision of solitons with $V = 2$, when the probe beam was initially guided by: (a) S1 and (b) S2. (c) Normalized confinement energy for each waveguide where the probe beam was initially guided by: S1 (square) and S2 (circle).



(a) (b)

Fig. 8. (a) Collision between three solitons: one with $V = 1$ and the other two with $V = 0$, separation between them of $c = 8$. (b) Behavior of the probe beam initially guided by the central soliton.

5. Conclusions

In this work we present a numerical study of the asymmetric collision of two fundamental (1 + 1)-D bright spatial solitons in a Kerr media and its waveguide properties. We show that is possible to control the amount of light confined in each waveguides photo-induced by the solitons changing the collision angle and separation between the solitons. The results demonstrate that for transversal velocities *V* less than 1, the probe beam energy confined by each waveguide after the collision, is independent of which soliton initially guided the probe beam. However, for *V* larger than 1, the amount of energy confined by each waveguide depended on which soliton initially guided the probe beam. It is demonstrated that as in the temporal case the collision of spatial solitons can be used as a mode separator when the probe beam is a combination of fundamental and high order modes. The results can be used as reference for other types of nonlinearities and to design more complicated soliton collision configurations in order to predict the waveguide properties of such elements.

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