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NOON states in entangled cavities

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ABSTRACT

We show how NOON states may be generated entangling two cavities by passing atoms through them. The atoms interact with each cavity via two-photon resonant transitions. We take advantage of the fact that depending on the state the atom enter (excite or ground), it leaves or takes two photons per interaction and leaves the cavities in a pure state.

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1. Introduction

A major effort has been directed towards the generation of nonclassical states of electromagnetic fields, in which certain observables exhibit less fluctuations (or noise) than in a coherent state, whose noise is referred to as the standard quantum limit (SQL). Nonclassical states that have attracted the greatest interest include (a) macroscopic quantum superpositions of quasiclassical coherent states with different mean phases or amplitudes, also called “Schrödinger cats” [1,2], (b) squeezed states [3] whose fluctuations in one quadrature or the amplitude are reduced beyond the SQL, (c) the particularly important limit of extreme squeezing, i.e. Fock or number states [4] and more recently, (d) nonclassical states of combined photon pairs also called NOON states [5,6]. It is well known that these multiphoton entangled states, can be used to obtain high-precision phase measurements, becoming more and more advantageous as the number of photons grows. Many applications in quantum imaging, quantum information and quantum metrology [7] depend on the availability of entangled photon pairs because entanglement is a distinctive feature of quantum mechanics that lies at the core of many new applications. These maximally path-entangled multiphoton states may be written in the form

$$|NOON\rangle_{a,b} = \frac{1}{\sqrt{2}}(|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b). \quad (1)$$

In the case of cavities [8–11], which we will study in this communication, this state contains N indistinguishable photons in an equal superposition of all being in cavity A or cavity B.

It has been pointed out that NOON states manifest unique coherence properties by showing that they exhibit a periodic transition between spatially bunched and antibunched states when undergo Bloch oscillations. The period of the bunching/antibunching oscillation is N times faster than the period of the oscillation of the photon density [12].

Most schemes to produce NOON states are in the optical regime [5,6]. In this contribution, we would like to analyze the microwave regime [8–11] where we will show how to generate NOON states in cavities, by passing atoms through them in such a way that the cavities get entangled. We consider two-photon resonant transitions and quality factors sufficiently high such that it allows us to neglect dissipative effects [4,13,14]. High quality factor cavities may be constructed in the microwave regime, with factors from 10^7 [9] to 10^{10} [11]. Rydberg atoms (usually ^{85}Rb atoms) with excited state of $40S_{1/2}$ and ground state of $39S_{1/2}$, may be passed through them, either to produce non-classical states and/or to measure field properties. Here we will show that atoms can entangle two cavities like the ones shown in Fig. 1, such that NOON states for microwave quantized stationary fields may be generated.

2. Two-photon dynamics

In this section we study the atomic behavior when light interacts with matter in a two-photon resonant transition. We consider the two-photon interaction Hamiltonian [4,11,14]

$$H_I^{(a)} = (\Delta + \chi a^\dagger a)\sigma_z + \lambda(a^2\sigma_+ + a^{\dagger 2}\sigma_-), \quad (2)$$

where λ is the coupling constant, a and a^\dagger are the annihilation and creation operators for the field mode (cavity A), respectively, $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$, are the Pauli spin-flip operators for the two-photon transitions, here $|e\rangle(|g\rangle)$ means excited (ground) atomic state.

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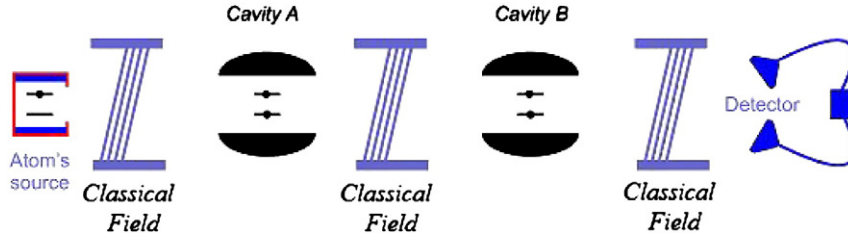


Fig. 1. Proposed experimental setup to produce NOON states in entangled cavities. An excited atom may pass cavities A and B and leave exactly 2 photons or get entangled with the cavities. The three classical fields are used to prepare the atoms either in ground, excited or superposition of ground and excited states.

We consider the intermediate state to be so far from resonance that it can be adiabatically eliminated to give an effective two-photon coupling of the above form. It contains a Stark shift that leads to an intensity dependent transition frequency. The Stark shift coefficient is denoted as χ and $\Delta = \omega_0 - 2\omega$ is the detuning, where ω_0 is the unperturbed atomic transition frequency and ω is the cavity field frequency. Knight and Shore [15] have investigated the validity of the adiabatic elimination for a single atom evolution.

The evolution operator is given by (in the atomic basis, see [16])

$$U_I^{(a)}(t) = e^{-iH_I^{(a)}t} = e^{i\frac{\chi}{2}t} \begin{pmatrix} C_{\hat{n}} & -iS_{\hat{n}}^{\dagger}a^2 \\ -ia^{\dagger 2}S_{\hat{n}} & C_{\hat{n}-2} \end{pmatrix}, \quad (3)$$

where

$$C_{\hat{n}} = \cos(\delta_{\hat{n}}t) - i\frac{\Gamma_{\hat{n}}}{\delta_{\hat{n}}} \sin(\delta_{\hat{n}}t), \quad S_{\hat{n}} = \lambda \frac{\sin(\delta_{\hat{n}}t)}{\delta_{\hat{n}}}, \quad (4)$$

with

$$\delta_{\hat{n}}^2 = \Gamma_{\hat{n}}^2 + \chi^2(\hat{n} + 1)(\hat{n} + 2), \quad \Gamma_{\hat{n}} = \frac{\Delta + \chi(\hat{n} + 1)}{2}, \quad (5)$$

and $\hat{n} = a^{\dagger}a$.

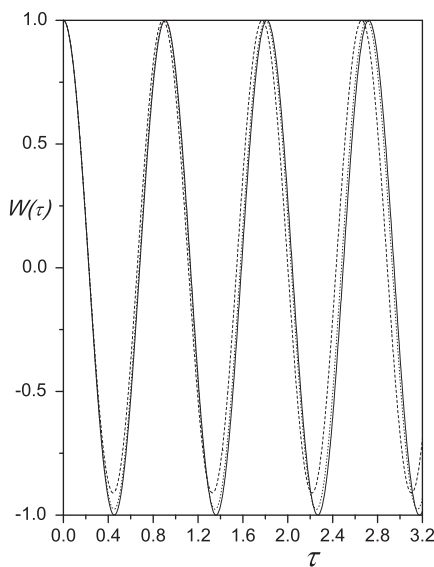


Fig. 2. Plot of the atomic inversion, the atom is initially in its excited state. The field is in the number state $|2\rangle_a$. Solid curve is for $\chi/\lambda = 0$ and $\Delta/\lambda = 0$, dashed curve is for $\chi/\lambda = 0.5$ and $\Delta/\lambda = 0$ and dot-dashed curve is $\chi/\lambda = 0$ and $\Delta/\lambda = -0.75$. The scaled time, τ is defined as $\tau = \lambda t$.

The evolution operator for the second cavity (B), $U_I^{(b)}$, reads

$$U_I^{(b)}(t) = e^{-iH_I^{(b)}t} = e^{i\frac{\chi}{2}t} \begin{pmatrix} C_{\hat{N}} & -iS_{\hat{N}}^{\dagger}b^2 \\ -ib^{\dagger 2}S_{\hat{N}} & C_{\hat{N}-2} \end{pmatrix}, \quad (6)$$

with $\hat{N} = b^{\dagger}b$, b the annihilation operator for cavity B. We assume that the Stark shift parameters, interaction constants and detunings are equal for both cavities. In Fig. 2 we plot the atomic inversion $W(\tau) = P_e(\tau) - P_g(\tau)$, where $P_e(\tau)$ ($P_g(\tau)$) is the probability to find the atom in its excited (ground) state, provided it enters cavity A initially in the excited state, as a function of the scaled time $\tau = \lambda t$. The initial state of the field is the number state $|2\rangle$. It is plotted for different Stark shift parameters and detunings. The figure shows that, at several times the atom is in its ground state, or close to it. In the rest of the paper we will look in particular at the time $\tau_p \approx 3.16$ as it is a time when the field gains two photons.

In case we consider the atom initially in its ground state, it may be shown that the probability to find it in the ground state at time τ_p is approximately zero. In this case the atom removes, in a clean form, two photons from the cavity.

2.1. Generation of the state $|2\rangle_a|2\rangle_b$

We can generate the state $|2\rangle_a|2\rangle_b$ if we start from two empty cavities and pass an initially excited atom through cavity A, let it interact with the vacuum field a time τ_p , then the atom leaves the cavity in its ground state (See Fig. 2). After it exits cavity A, a classical field is turned on (second classical field in Fig. 1), which produces a rotation in the atom that takes it again to its excited state. Then it passes through cavity B leaving again two photons in it as it exits (of course for the same interaction time τ_p). In this way we pass from the state $|0\rangle_a|0\rangle_b$ to $|2\rangle_a|2\rangle_b$.

3. NOON states by entangling the cavities

We now consider the atom to be in a superposition of its ground and excited states, i.e.

$$|\psi_{atom}\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (7)$$

Both cavity fields have been prepared in the state $|2\rangle$. We now consider the ideal case of $\chi = 0$ and $\Delta = 0$, after interaction with cavity A, we obtain (approximately) the field-atom entangled state

$$|\psi_{a-f}\rangle = \frac{i}{\sqrt{2}} \begin{pmatrix} |0\rangle_a \\ |4\rangle_a \end{pmatrix}, \quad (8)$$

without disturbing the atom, it passes now through the second cavity, in the number state $|2\rangle_b$, that produces the field-atom-field entangled state

$$|\psi_{f-a-f}\rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} |4\rangle_a|0\rangle_b \\ |0\rangle_a|4\rangle_b \end{pmatrix}. \quad (9)$$

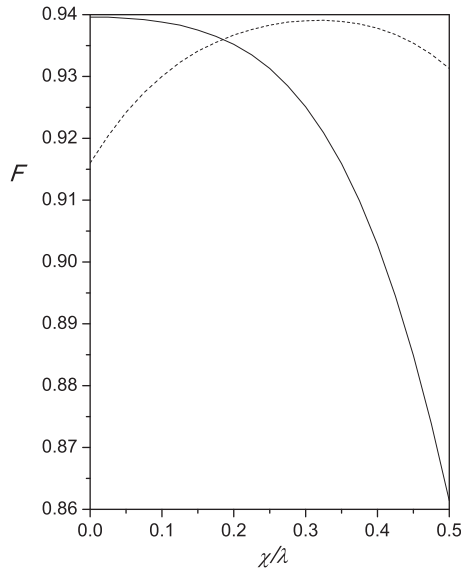


Fig. 3. Fidelity as a function of the Stark shift parameter. Solid line for $\Delta = 0$ and dashed line for $\Delta = -0.75\lambda$.

When the atom exits cavity B, the last classical field is turn on, rotating the atom, and therefore producing the state

$$|\psi_{f-a-f}\rangle = -\frac{1}{2} \left(|4\rangle_a |0\rangle_b - |0\rangle_a |4\rangle_b \right). \quad (10)$$

Finally, by detecting the atom in its ground state, the wave function is collapsed to entangled states of the two separate cavities, i.e. to the NOON state

$$|\psi_{f-f}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_a |4\rangle_b + |4\rangle_a |0\rangle_b). \quad (11)$$

Of course, detection of the atom in its excited state would have produced a NOON state with a different sign in between.

3.1. Fidelity

We now calculate the fidelity [17]

$$F = |{}_{a,b}\langle \text{NOON} | \psi_{f-f}(\tau_p) \rangle|^2 \quad (12)$$

which measures the “closeness” of the two quantum states $|\psi_{f-f}(\tau_p)\rangle$ and $|\text{NOON}\rangle_{a,b}$. We find $|\psi_{f-f}(\tau_p)\rangle$ by applying the evolution operators, (3) and (6), to the state $|2\rangle_a |2\rangle_b \frac{1}{\sqrt{2}} (|e\rangle + |b\rangle)$. We plot this function in Fig. 3, where we note a fidelity value $F \approx 0.94$ for the Stark shift parameter equal to zero. As this parameter increases the fidelity decreases but remains close to its initial value. Moreover, it may be increased for non-zero values of the Stark shift parameter by properly adjusting the detuning such that it cancels out the effect of the AC Stark effect. It is clear that dissipative effects would produce the fidelity also to decrease, as it is a well-known fact that dissipation erases non-classical features.

4. Conclusion

It has been shown that by controlling the interaction time between of atoms passing through two cavities, cavity fields may be entangled. A final detection of the atom in its excited or ground state yields a NOON state, i.e. an entangled state of both cavities. In this form we have added another system in which NOON states may be produced, i.e. extended the regime to microwave cavities. Measurement of cavity fields may be done via quantum state reconstruction even in the presence of dissipation [18].

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