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Alternative (1+1)-D Dark Spatial Soliton-Like Distributions in Kerr Media

D. Ramírez Martínez[1], M.M. Méndez Otero,[1] M.L. Arroyo Carrasco[1] and M.D. Iturbe Castillo[2]

1. Benemérita Universidad Autónoma de Puebla, Avenida San Claudio y Río Verde, colonia San Manuel, Ciudad Universitaria, Puebla, Puebla, C.P. 72570, México

2. Instituto Nacional de Astrofísica, Óptica y Electrónica, Luis Enrique Erro#1, Tonantzintla Puebla, C.P. 72840, México

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Abstract: In this paper, we solve numerically the nonlinear Schrödinger equation for a defocusing Kerr media with initial conditions different to the hyperbolic tangent function. We demonstrate that initial conditions with a similar position dependence than the hyperbolic tangent evolve to a hyperbolic tangent function, therefore can be considered as (1+1)-D spatial solitons. The waveguide induced by these conditions is not single mode; it has the capacity to confine one mode more.

Key words: Kerr effect, solitons optical, optical waveguide nonlinear.

1. Introduction

It is well accepted that spatial solitons are beams that create their own waveguide [1, 2]. This kind of beam can be generated when some specific field distribution propagates in an intensity-dependent refractive-index medium. The equation that described the changes in amplitude of a field in such media is known as the nonlinear Schrödinger equation (NLSE). Exact solutions to the NLSE are named as fundamental solitons. The soliton can be seen as a beam where compensation between nonlinearity and diffraction exists. The idea of using spatial solitons to guide other beams was suggested theoretically by Chiao et al. [3], and proved experimentally by Luther-Davies and Yang [4]. Collision of solitons was also suggested in order to control a weak beam [5-7]. The use of spatial solitons as optical channels for probe beams has a direct effect on the development of logical and interconnecting devices [8, 9].

Experimentally, it can be difficult to have field distributions that correspond to the exact solutions of the NLSE in order to generate dark spatial solitons [10]. In this paper we present two initial conditions for a negative Kerr media that evolve to hyperbolic tangent function after propagation. The waveguide properties exhibited by these conditions are very similar to that of the fundamental dark soliton.

In the next section we present the equations that are going to be solved numerically and the waveguide property of the fundamental dark soliton is analyzed. Then the new initial conditions are presented and propagated. The waveguide properties of the new solution are analyzed in its dependence with the initial relative position between the soliton and the probe beam. Finally conclusions are given.

2. Theory

We are going to use the same set of equations presented in Ref. [8], where the simultaneous propagation of an intense (q_1) and a probe (q_2) beam was described in a Kerr media:

$$i\frac{\partial q_1}{\partial Z} = \frac{1}{4}\frac{\partial^2 q_1}{\partial X^2} \pm \frac{L_D}{L_{NL}}|q_1|^2 q_1 \qquad (1)$$

Corresponding author: D. Ramírez Martínez, master, research field: nonlinear optics. E-mail: darama@hispavista.com.

$$i\frac{\partial q_2}{\partial Z} = \frac{1}{4} r_n r_k \frac{\partial^2 q_2}{\partial X^2} \pm \frac{2L_D}{r_k L_{NL}} |q_1|^2 q_2 \qquad (2)$$

where q_j is the amplitude of the optical field normalized to the maximum intensity I_{mj}, $L_D = n_0 k_0 x_0^2 / 2$ is the diffraction length with n_0 the linear refractive index, k_0 is the wavenumber, x_0 is the initial beam width and $L_{NL} = 1/|n_2| k_0 I_m$ is the nonlinear length with n_2 the nonlinear refractive index, $r_n = n_{01} / n_{02}$ and $r_k = k_{01} / k_{02}$ where n_{0j} are the linear refractive index, k_{oj} are wave number where the subscript j is 1 for soliton and 2 for the probe beam respectively. In this work, by simplicity we utilized $r_n = r_k = 1$, this means that the soliton and the probe beam have the same wavelength and direction. It is possible to use values different from 1, which depends of the wavelengths used and the refractive index presented by the selected material, however, the main characteristics of the propagation can be obtained with a value of 1. This set of equations with a different normalization was used in Ref. [8] to analyze the collision of two bright spatial soliton and its waveguide properties.

Eq. (1) is the nonlinear Schrödinger equation, which has been studied analytically by the inverse scattering method and is well-known that admits two stationary solutions soliton type, which are:

$$q(X, Z) = \sec h(\sqrt{2} X) \exp(-iZ/2) \qquad (3)$$

and

$$q(X, Z) = \tanh(\sqrt{2} X) \exp(-iZ) \qquad (4)$$

where $q_1 = q$. These solutions are known as the bright and dark solitons, for a positive and negative nonlinear refractive index material, respectively. To investigate the behavior of the probe beam guided by the photoinduced waveguide of the soliton we considered a Gaussian beam with the following dependence:

$$q_2(X, Z) = \exp(-(\frac{X-h}{w})^2) \qquad (5)$$

where h is the separation between the soliton and the probe beam and w is the initial width of the beam with normalized amplitude q_2.

3. Results

The propagation of a fundamental dark soliton, given by Eq. (4), is shown in Fig. 1, where we can see that there is not modification of the distribution along the propagation distance. Considering a probe beam with small amplitude and the same wavelength than the soliton, the coupled Eqs. (1) and (2) were solved for different separations h between the soliton and the probe beam. Both beams are propagated in the same direction. Typical results for the evolution of the probe beam for different values of h are presented in Fig. 2. For $h = 0$, that is, when the probe beam and the soliton are fully overlapped, the waveguide confines totally the probe beam as shown in Fig. 2a. The propagation distance was of 20 diffractions lengths.

As it was waited, when h was increased, the amount of energy of the probe beam confined in the waveguide photoinduced by the soliton beam was reduced. In some cases the probe beam realized some oscillations both in amplitude and position around of center of the soliton as shown in Figs. 2b and 2d. This effect is due to the fact that the photoinduced refractive index by the soliton changes gradually and the probe beam bounce along it. For values of $h > 1.5$, a minimum in the intensity profile of the probe beam was obtained in the position corresponding to the soliton (Figs. 2e-2h). A similar result was previously reported in Ref. [11],

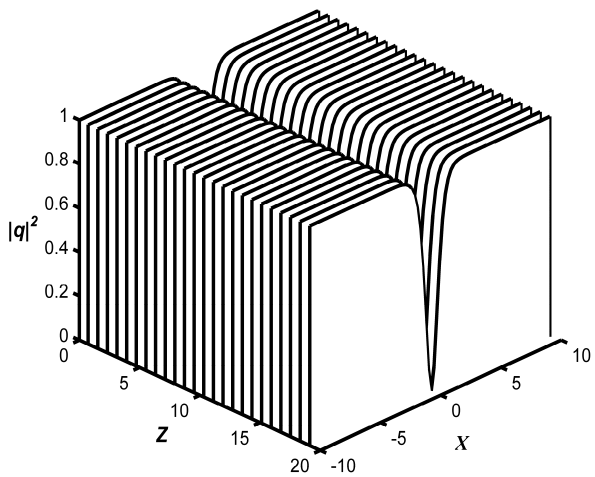

Fig. 1 Propagation of a fundamental dark soliton to 20 diffraction lengths.

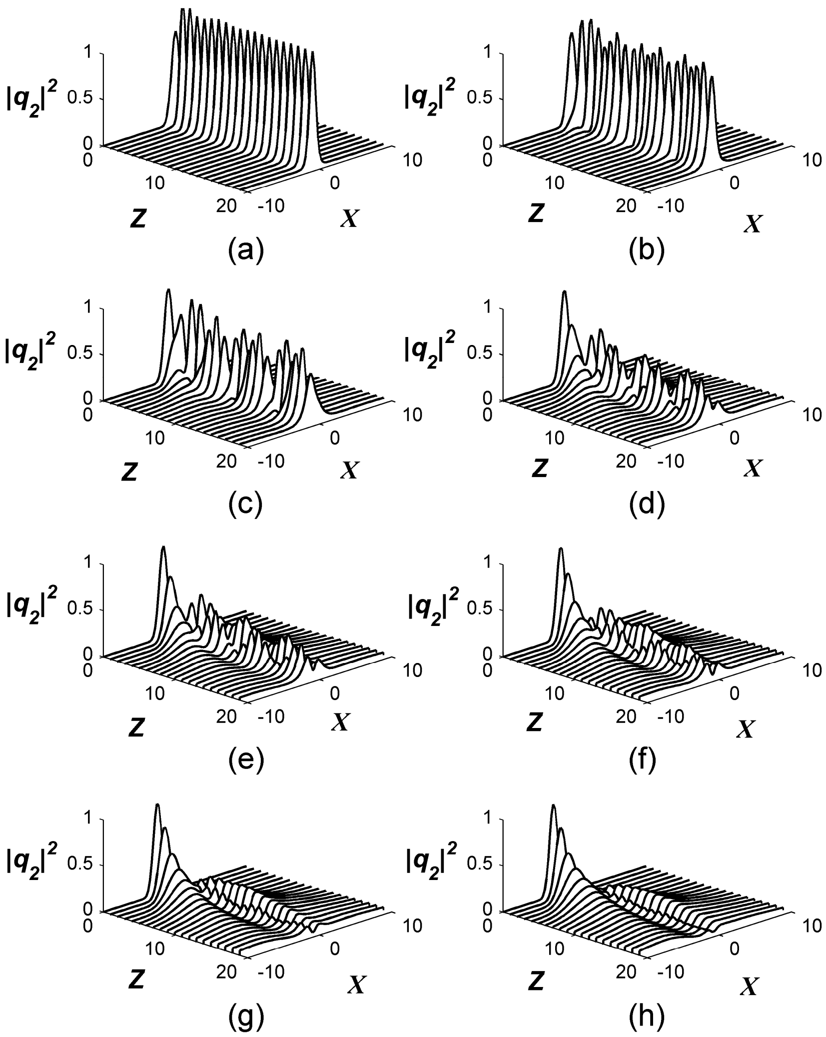

Fig. 2 The propagation of a Gaussian probe beam with *w* = 0.9, for different distances *h* from the soliton: (a) *h* = 0, (b) *h* = 0.5, (c) *h* = 1, (d) *h* = 1.5, (e) *h* = 1.7, (f) *h* = 2, (g) *h* = 2.5, (h) *h* = 3.

where an experimental analysis of trapping efficiency was made for different launching angles between the soliton and the probe beam. This behavior was called "leaky mode" and was explained as the excitation of high order modes in the waveguide. However, no analytical expression was suggested for such mode. Larger separations between the soliton and the probe beam demonstrate that only a small portion of the probe beam was confined in the photoinduced waveguide. In Figs. 3a and 3b, the propagation distance was increased in order to demonstrate that some portion of the probe beam is confined but did not have the incident Gaussian distribution and in the central position of the waveguide there is a minimum in intensity of the probe beam. It is important to mention that the previous results are the same if the analysis is made with a fundamental bright soliton to photoinduced the waveguide, i,e, the same dependence of the probe beam with separation is obtained for materials with positive nonlinearity.

The behavior obtained for the probe beam suggested that the waveguide induced by the soliton has the property to confine another field distribution different from the Gaussian. The behavior obtained in Figs. 2d-2f seems to be the interference of a Gaussian mode with a high order mode. To verify this assumption, we

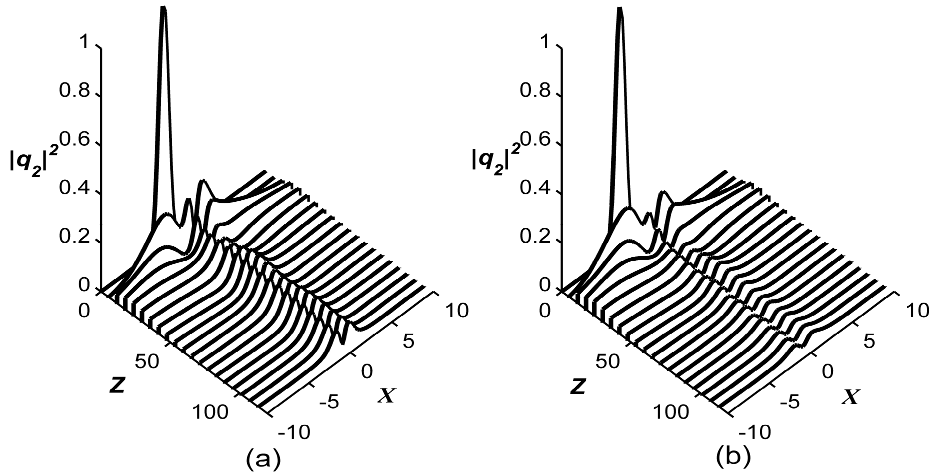

Fig. 3 The propagation of a Gaussian probe beam with *w* = 0.9, for (a) *h* = 2.5, (b) *h* = 3 to 120 diffraction lengths.

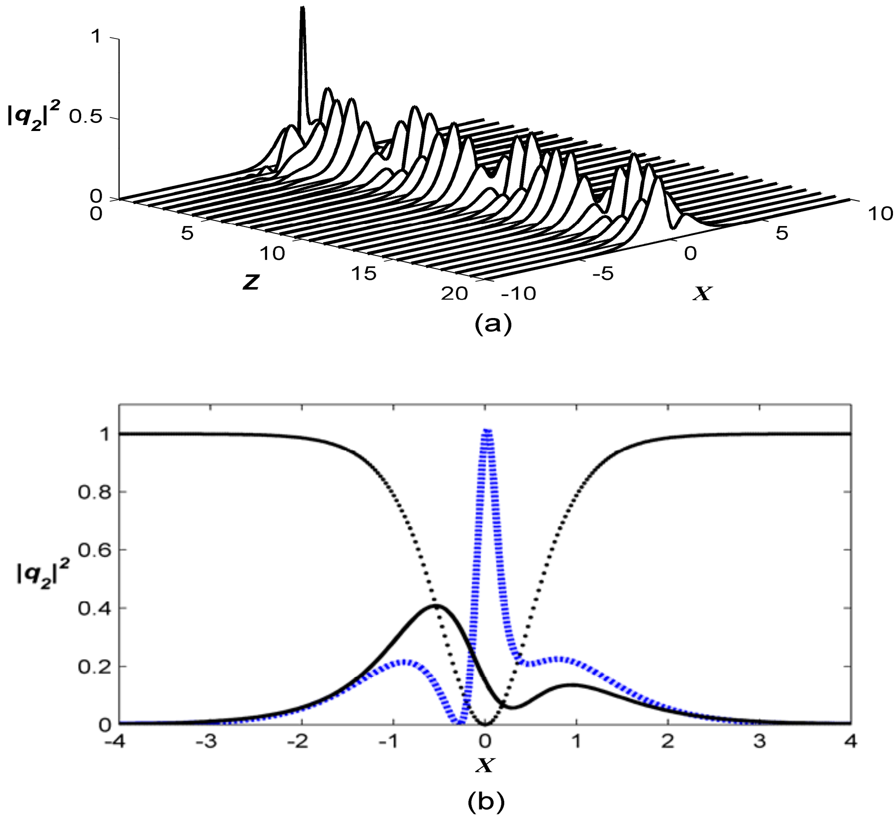

Fig. 4 (a) Evolution of the initial condition given by Eq. (6) as probe beam with A = 7 and (b) its intensity profiles at the beginning (dashed), and end of the propagation (solid line) and profile for soliton (solid circles).

performed the propagation of the soliton with the following initial condition for the probe beam:

$$q_2(X,Z) = \exp(-X^2) + X\exp(-AX^2). \quad (6)$$

We obtain a behavior as that shown in Fig. 4 that is similar to that observed in Figs. 2d-2f.

However, some part of the light was not confined to the waveguide. This result suggests the possibility that the next mode was similar to a Hermite-Gauss beam. To verify this, we utilized as a probe beam the following field distribution:

$$q_{21}(X,Z) = X\sec h(BX) \quad (7)$$

When the soliton and the probe beam given by Eq. (7)

were propagated, a very well confined probe beam was obtained, without loss in intensity and without change in its intensity profile (Fig. 5).

4. New Soliton-Like Distributions

Although a mathematical algorithm to find new solutions of the NLSE exists [12], we look for functions that present a similar amplitude dependence with the position than the exact solutions For the negative nonlinearity we look for functions with a change in sign in the amplitude for positive and negative values of the transversal coordinate. Two functions were found to fill such requirement:

$$q(X,Z) = \frac{2}{\pi}\tan^{-1}(cX) \tag{8}$$

$$q(X,Z) = \frac{X}{\sqrt{X^2+d}}. \tag{9}$$

where c and d are constants related to the width of the functions.

Eqs. (8) and (9) are compared with the field distribution of a fundamental dark soliton in Fig. 6. We can observe that the distributions differ in their width and how they tend to values far from zero.

For the negative nonlinearity, the evolution of initial condition given by Eqs. (8) and (9) is shown in Figs. 7 and 8, respectively. We can observe how at the beginning of the propagation there is the formation of two gray channels with small amplitude that propagated away the center, leaving at the center a fundamental dark soliton. The behavior observed for both initial conditions is similar to that obtained when the initial width of the fundamental soliton is larger than the adequate, giving rise to a pair of gray solitons [13]. However, changing width of the initial condition given by the Eqs. (8) and (9) was not possible to obtain a behavior similar to that obtained for a fundamental soliton without the two gray solitons beside.

The waveguide properties of the new soliton-like

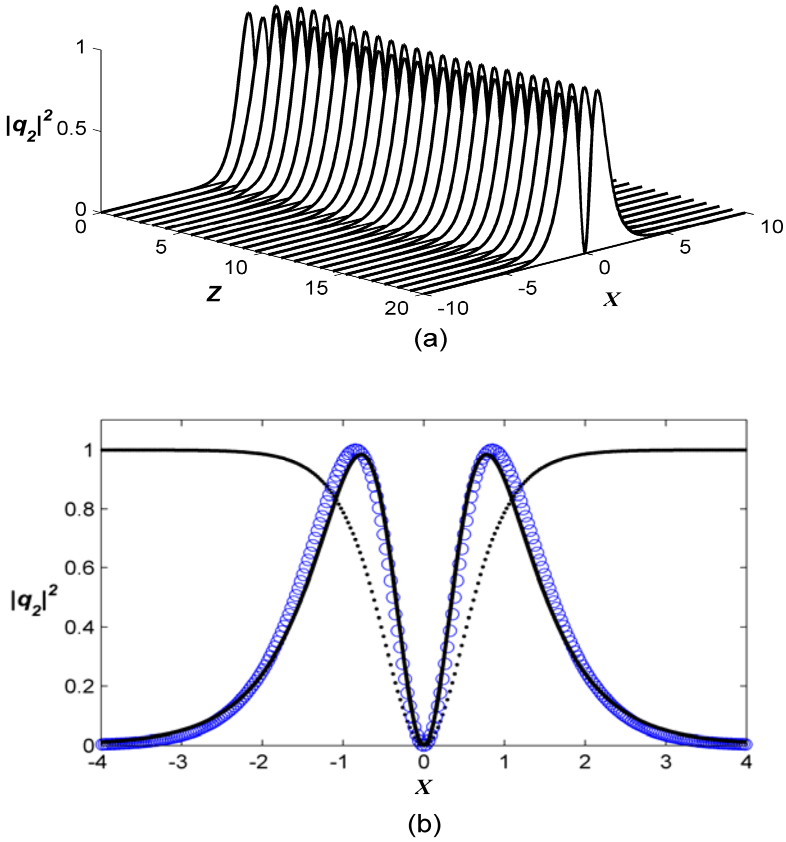

Fig. 5 (a) Propagation of an initial condition given by Eq. (7) for the probe beam with B = $(2)^{1/2}$, (b) input (open circles) and output profiles (solid line) for probe beam and profile for soliton (solid circles).

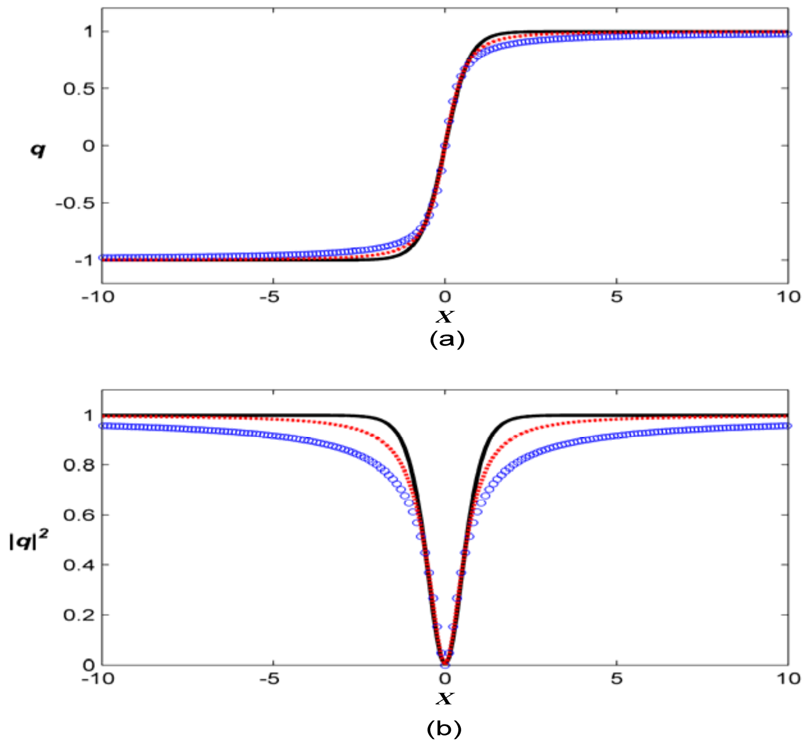

Fig. 6 **(a) Field amplitude; (b) intensity profiles, for a hyperbolic tangent (solid line), Eq. (8) with *c* = 3 (open circles) and Eq. (9) with *d* = 0.4 (solid circles).**

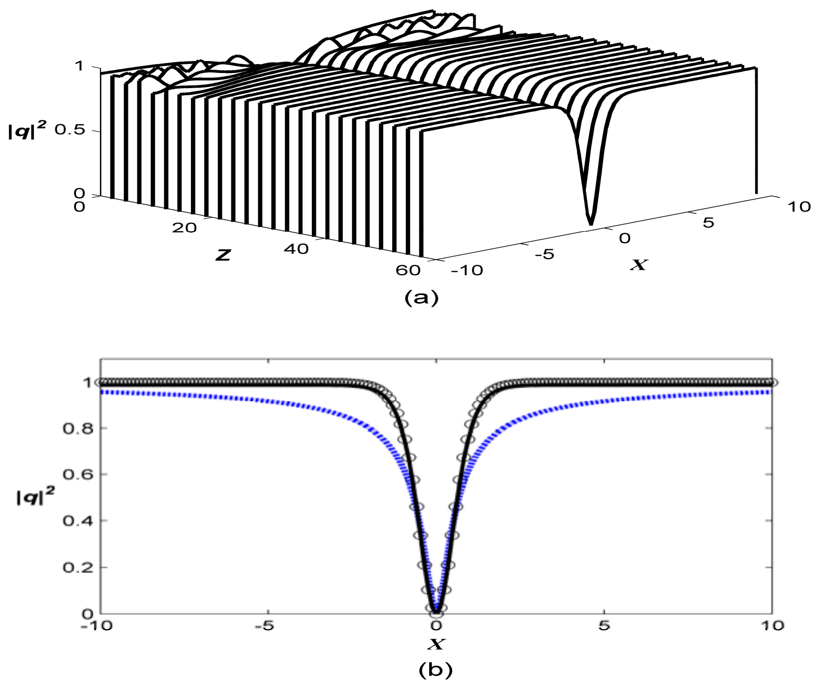

Fig. 7 **(a) Propagation; and (b) intensity profiles of input (dashed) and output (solid line) for the analytical solutions given by Eq. (8) with *c* = 3 and profile for soliton (open circles).**

distributions are similar to that obtained for the fundamental dark soliton in the confinement of the probe beam by the central waveguide. The two gray solitons presented a small confinement of the light that was not comparable to that captured by the central channel, as it can be seen in Figs. 9 and 10.

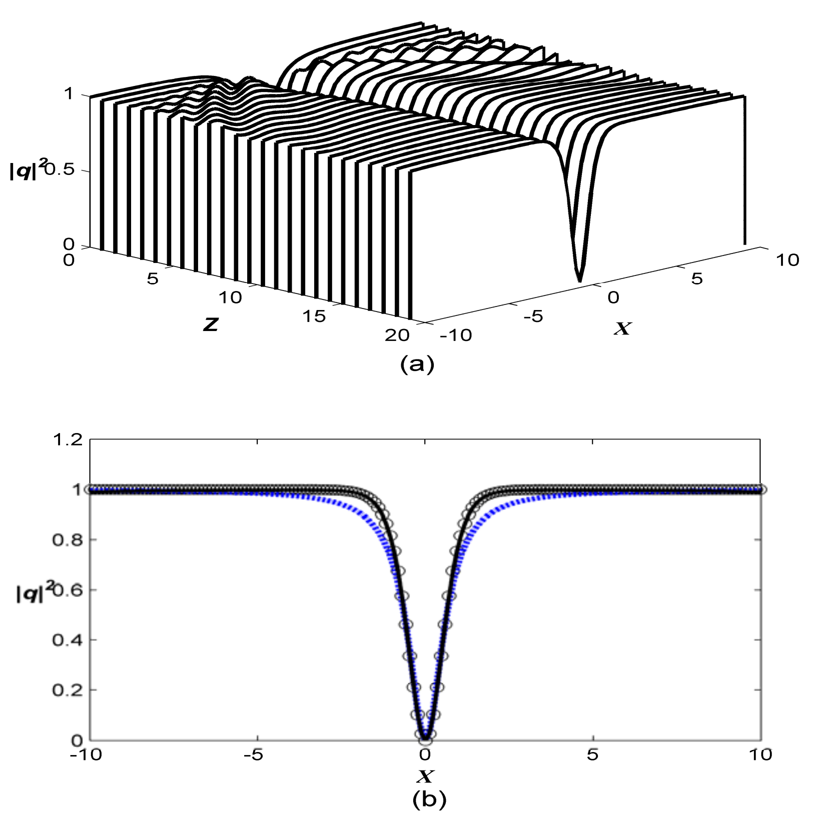

Fig. 8 (a) Propagation; and (b) intensity profiles of input (dashed) and output (solid line) for the analytical solutions given by Eq. (9) with *d* = 0.4 and profile for soliton (open circles).

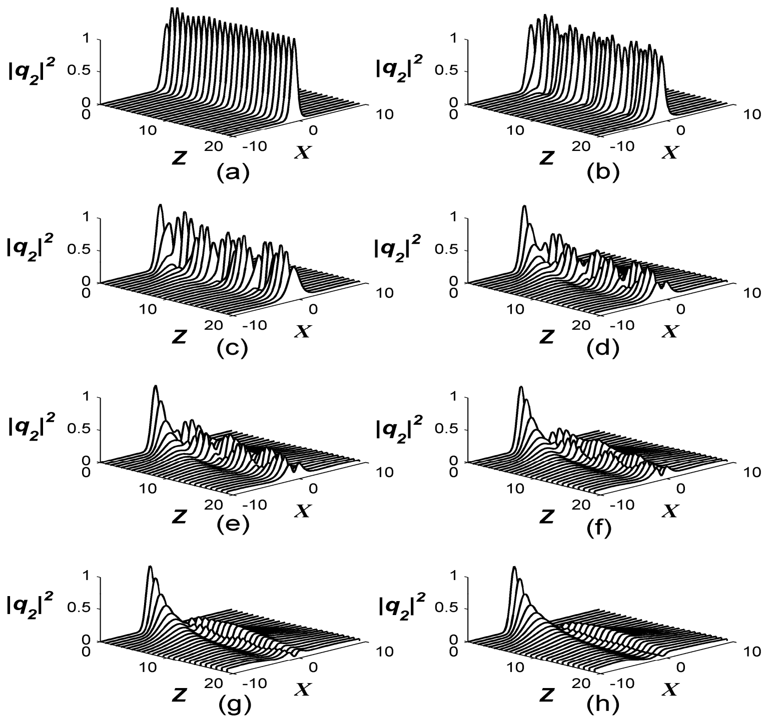

Fig. 9 Waveguide properties of the initial condition given by Eq. (9) when the probe beam is separated to a distance *h* from the soliton:(a) *h* = 0, (b) *h* = 0.5, (c) *h* = 1, (d) *h* = 1.5, (e) *h* = 1.7, (f) *h* = 2, (g) *h* = 2.5, (h) *h* = 3.

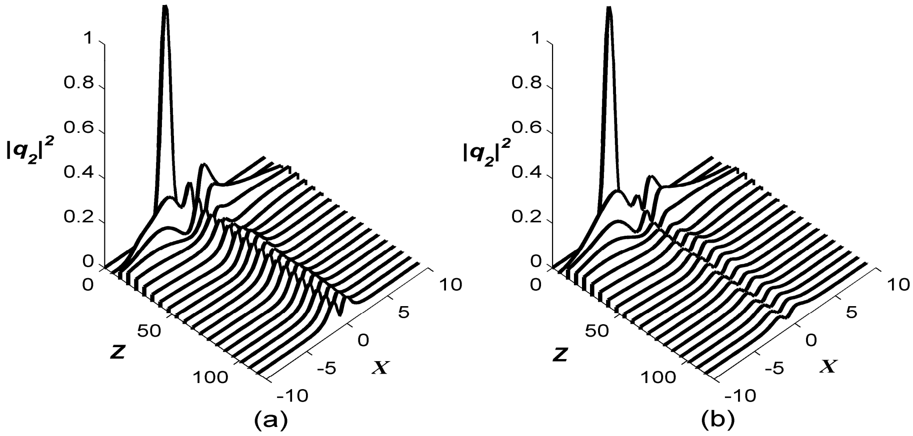

Fig. 10 Waveguide properties of the initial condition given by Eq. (9) when the probe beam is separated to a distance *h* from the soliton: (a) *h* = 2.5, (b) *h* = 3 to 120 diffraction length.

5. Conclusions

We present a numerical study of waveguide properties of a fundamental (1+1)-D dark spatial soliton in a Kerr material, considering as a probe beam a Gaussian distribution and different distances between this beam and the soliton. Two field distributions for a negative nonlinearity are proposed and propagated to demonstrate that they evolved to a fundamental dark spatial soliton. The waveguide properties of such soliton-like distributions were analyzed obtaining the same behavior than the fundamental dark soliton.

The results demonstrate that the waveguide photoinduced by the soliton is not single mode. The analytical expression of the high order mode confined by the waveguide was given.

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